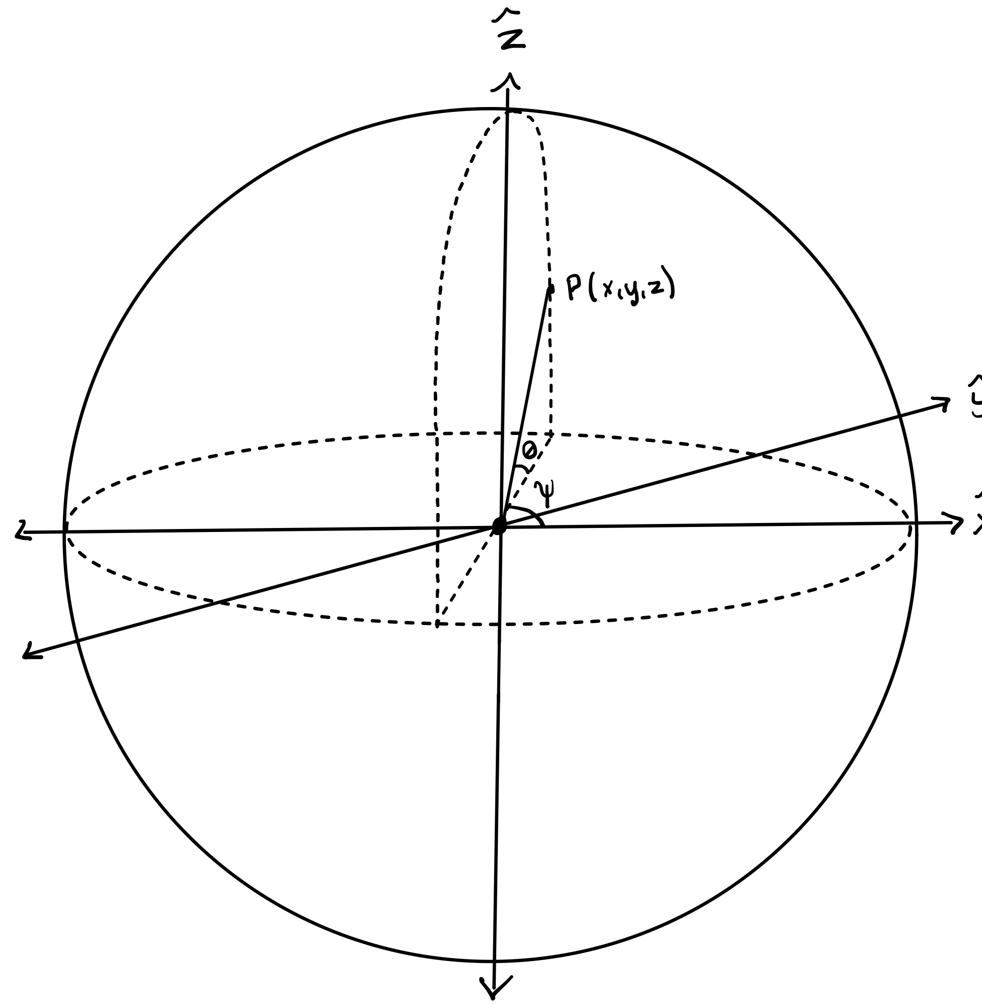
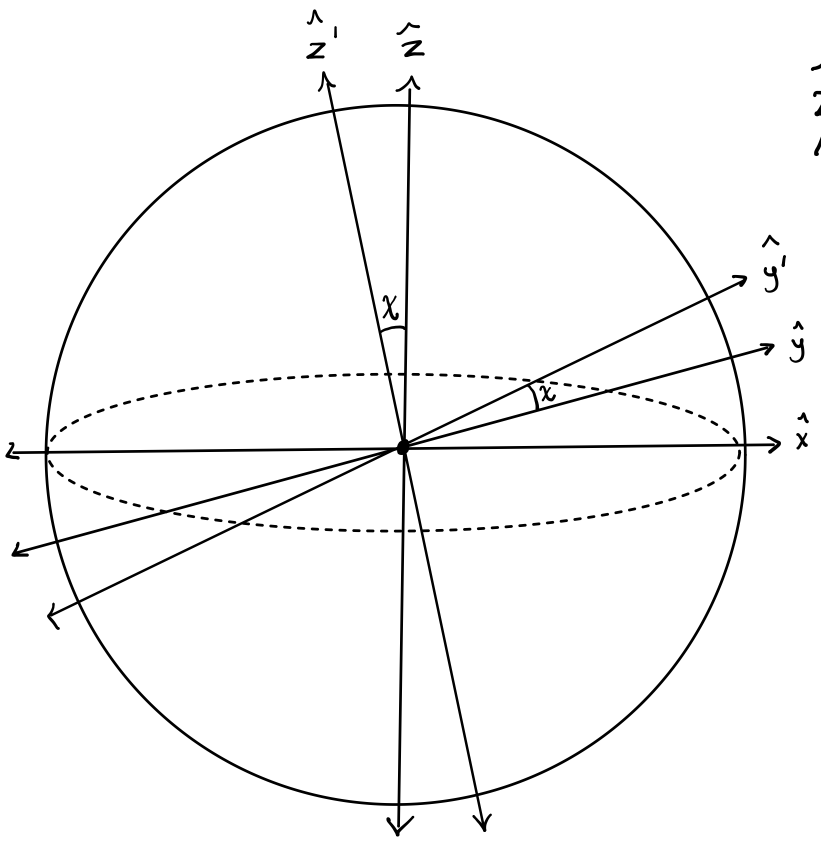


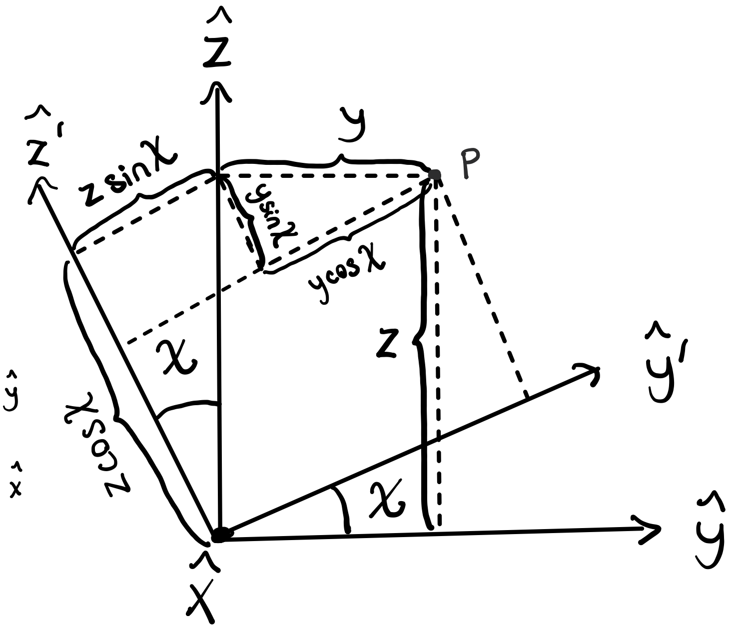
Cartesian Function

$$f(x, y) = \sqrt{r^2 - x^2 - y^2} = z$$

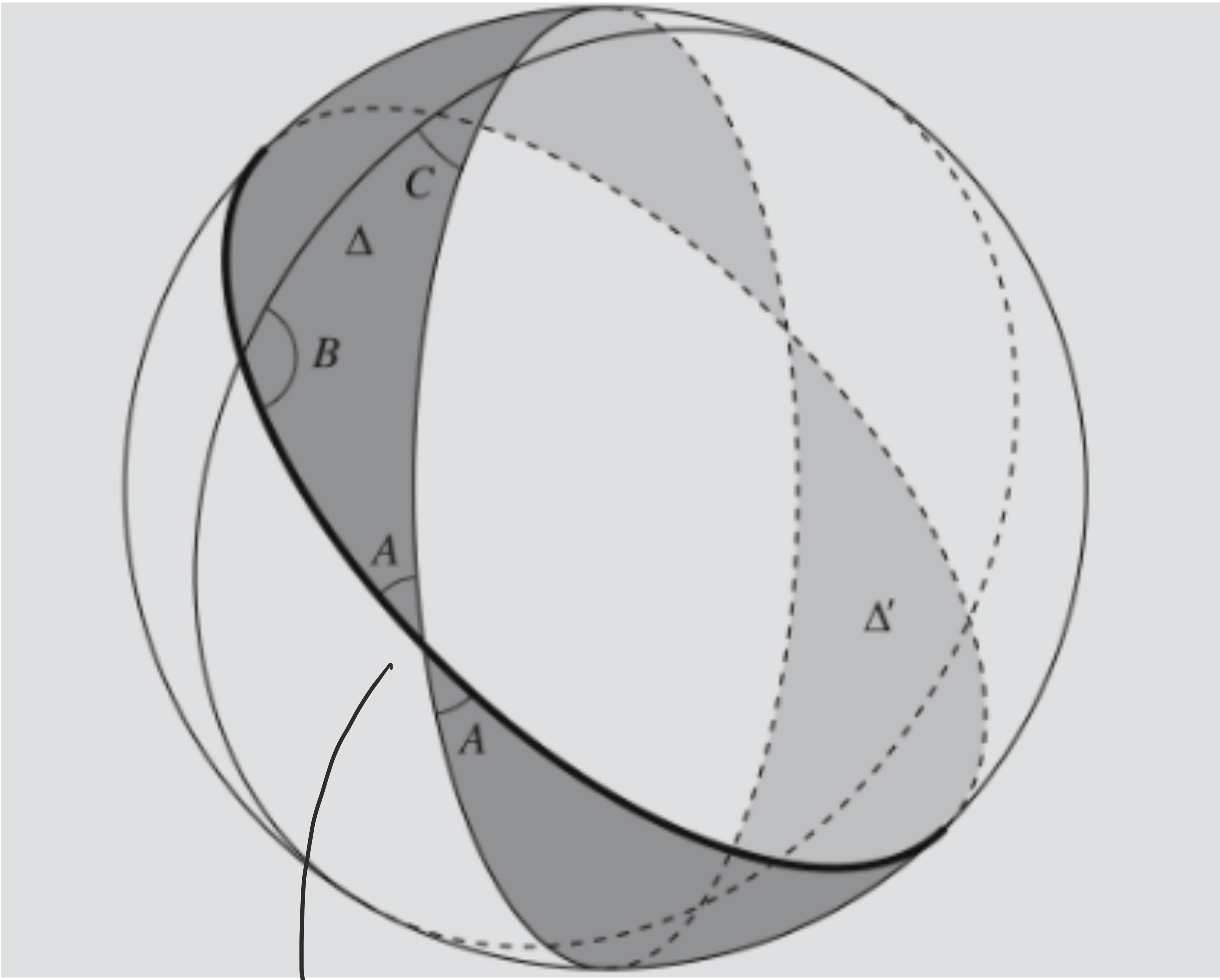
Polar coordinates

$$x = r\cos\psi\cos\theta$$
$$y = r\sin\psi\cos\theta$$
$$z = r\sin\theta$$





$$x' = x$$
$$y' = y\cos\chi + z\sin\chi$$
$$z' = z\cos\chi - y\sin\chi$$

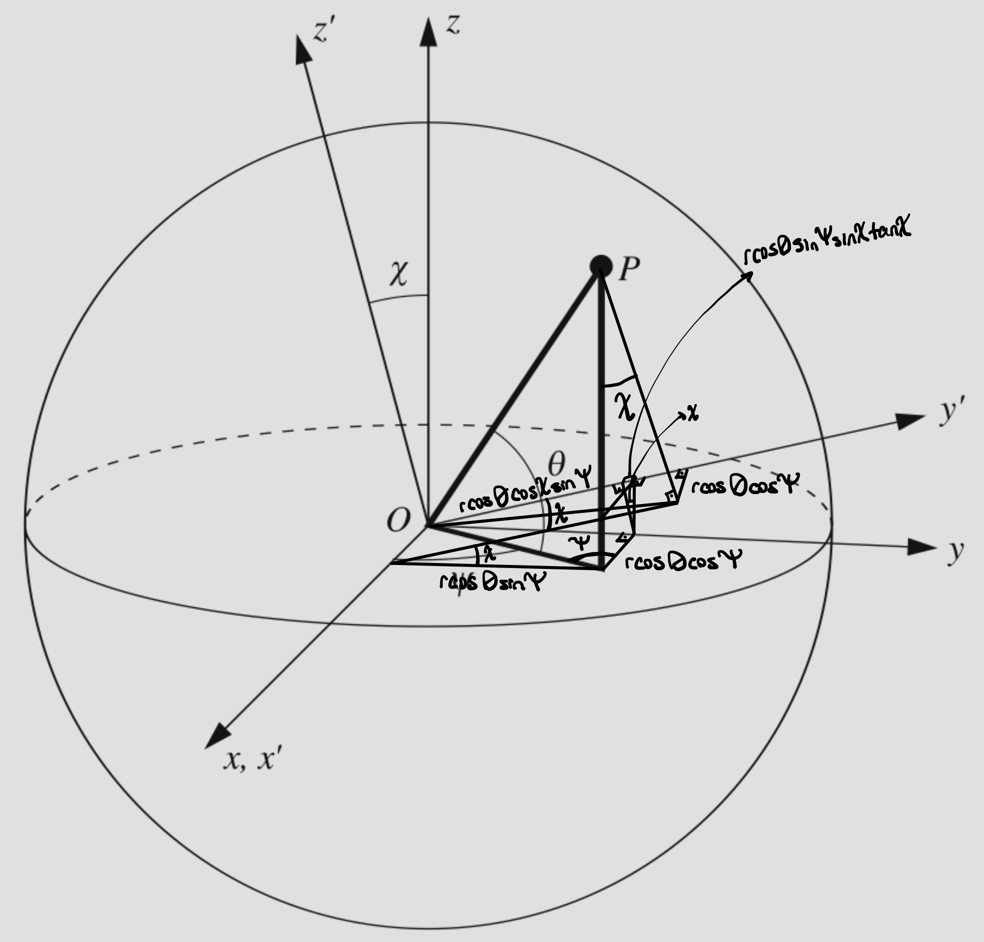


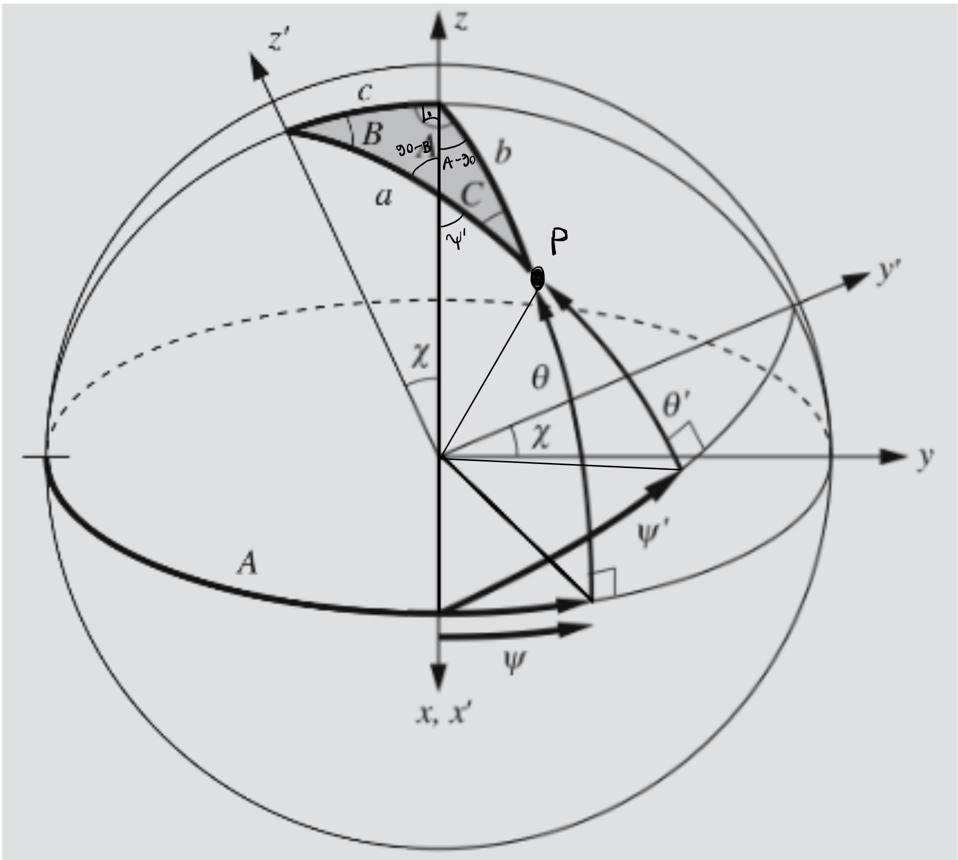
$$\left.\begin{aligned}
4\pi r^2 \cdot \frac{2A}{2\pi} &= 4AR^2\\
4\pi r^2 \cdot \frac{2B}{2\pi} &= 4BR^2\\
4\pi r^2 \cdot \frac{2C}{2\pi} &= 4CR^2
\end{aligned}\right\} 4R^2(A+B+C) = 4\pi R^2 + 4\,\text{Area}(\widehat{ABC}) \Longrightarrow \text{Area}(\widehat{ABC}) = (A+B+C-\pi)r^2 = E r^2$$

$$\begin{aligned}
x' &= x\\
y' &= y\cos\chi + z\sin\chi\\
z' &= z\cos\chi - y\sin\chi
\end{aligned}
\quad \text{and} \quad
\begin{aligned}
x &= r\cos\psi\cos\theta\\
y &= r\sin\psi\cos\theta\\
z &= r\sin\theta
\end{aligned}
\quad \text{and} \quad
\begin{aligned}
x' &= r\cos\psi'\cos\theta'\\
y' &= r\sin\psi'\cos\theta'\\
z' &= r\sin\theta'
\end{aligned}$$

$$\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

$$\begin{aligned}
\cos\psi'\cos\theta' &= \cos\psi\cos\theta\\
\sin\psi'\cos\theta' &= \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi\\
\sin\theta' &= \sin\theta\cos\chi - \sin\psi\cos\theta\sin\chi
\end{aligned}$$





$\psi = A - 90$

$\psi' = 90 - B$

$\chi = c$

$\theta = 90 - b$

$\theta' = 90 - a$

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = \sin\theta\cos\chi - \sin\psi\cos\theta\sin\chi$$

and

$$x' = x$$
$$y' = y\cos\chi + z\sin\chi$$
$$z' = z\cos\chi - y\sin\chi$$

$$\cos(90-B)\cos(90-a) = \cos(A-90)\cos(90-b) = \sin B \sin a = \sin A \sin b$$

$$\sin(90-B)\cos(90-a) = \sin(A-90)\cos(90-b)\cos(c) + \sin(90-b)\sin(c) = -\cos A \sin b \cos c + \cos b \sin c$$

$$\sin(90-a) = \sin(90-b)\cos(c) - \sin(A-90)\cos(90-b)\sin(c) = \cos A \sin b \sin c + \cos b \cos c$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

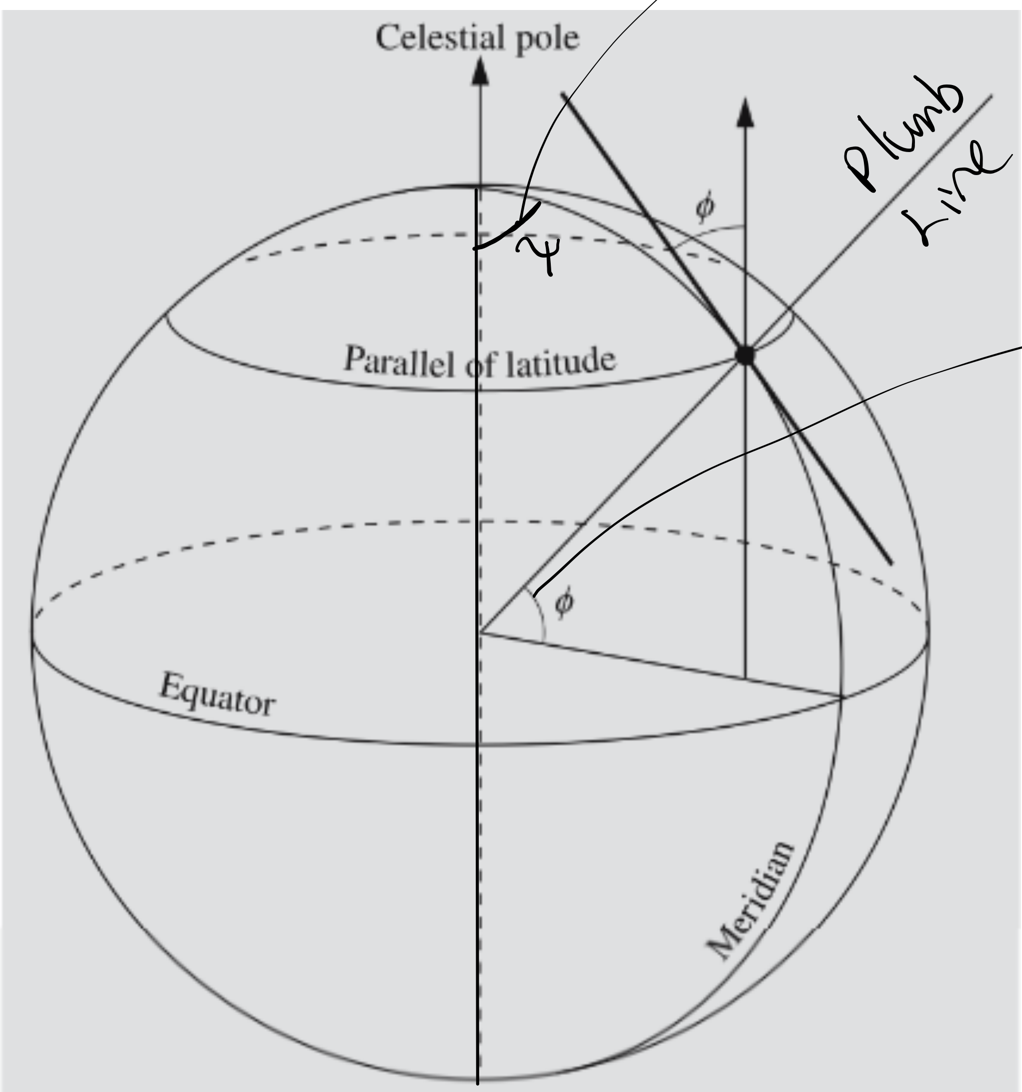
If we take the limit, letting the sides a, b and c shrink to zero, the spherical triangle becomes a plane triangle. If all angles are expressed in radians, we have approximately

$$\sin a \approx a\,, \quad \cos a \approx 1 - \frac{1}{2}a^2\,.$$

Substituting these approximations into the sine formula, we get the familiar sine formula of plane geometry:

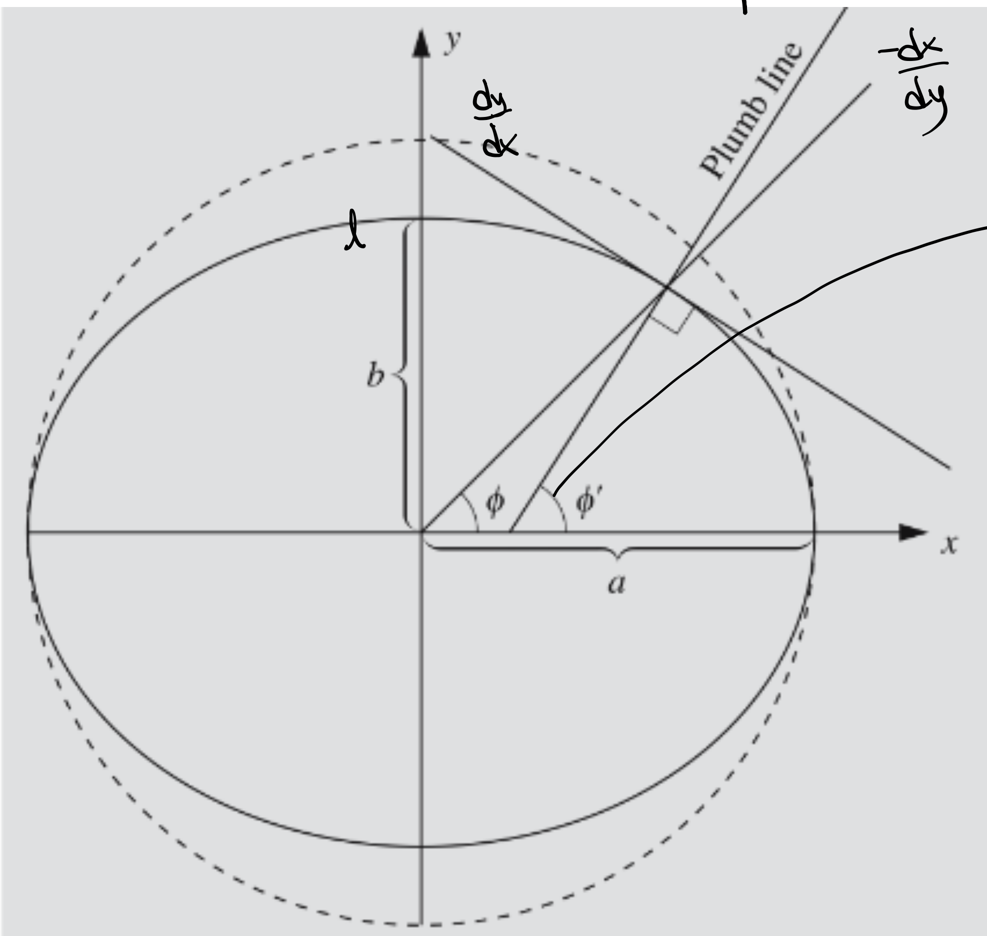
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\,.$$

geographical longitude (positive at east, negative at west)



geographical latitude (positive in north, negative in south)

Earth is an Oblate Spheroid



Geodetic Latitude (< Geocentric Latitude)

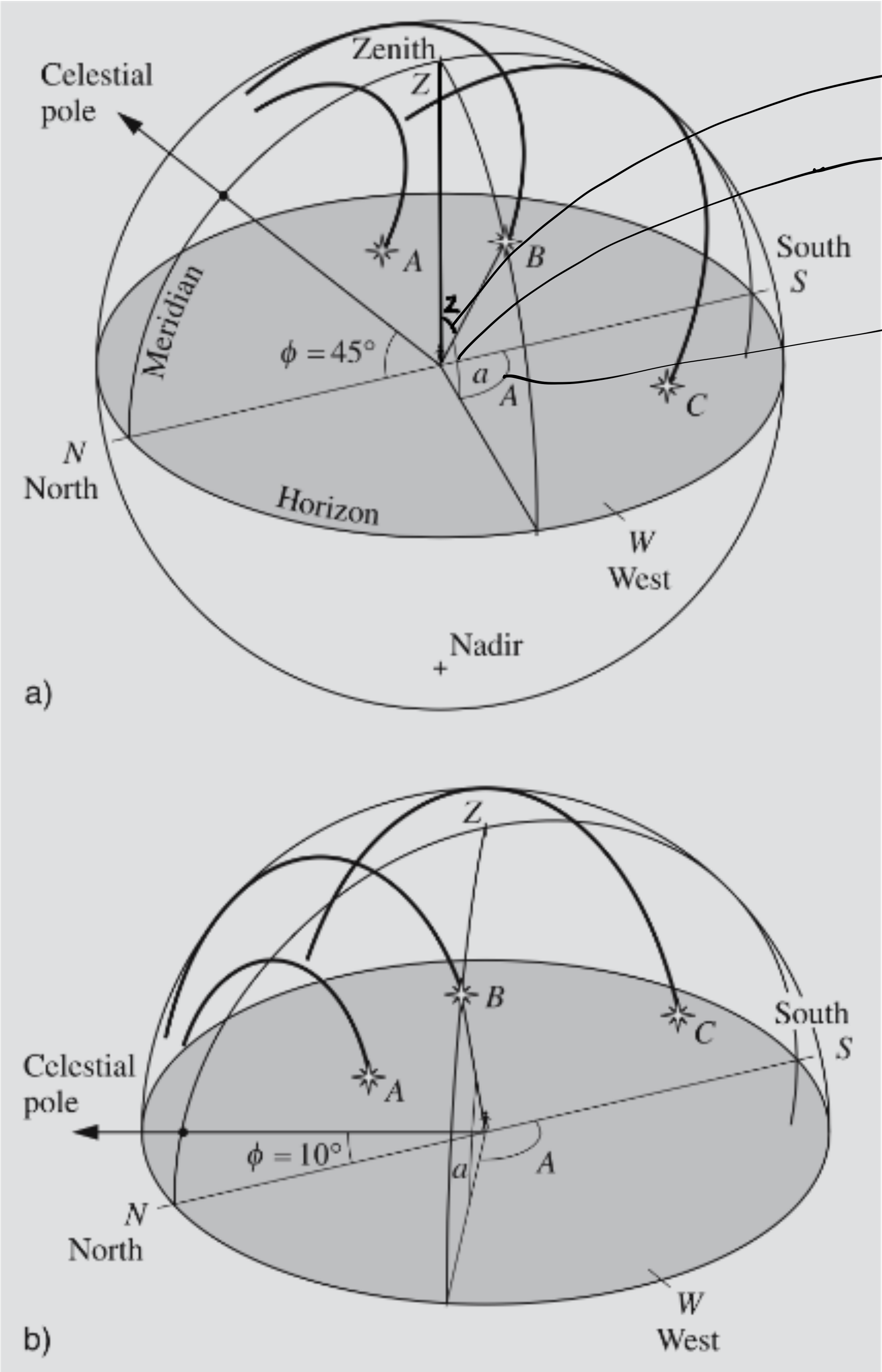
$$\tan\phi = -\frac{dx}{dy} = \frac{a^2}{b^2}\frac{y}{x} \Longrightarrow \tan\psi = \frac{y}{x} \text{ for a sphere}$$

$$\left(\frac{dy}{dx}\cdot\frac{-dx}{dy} = -1\right)$$

$$\tan\phi' = \frac{y}{x} \quad \left(\text{geocentric latitude}\right) \Longrightarrow \tan\phi = \frac{a^2}{b^2}\frac{y}{x} = \frac{a^2}{b^2}\tan\phi'$$

$$\frac{b^2}{a^2}\tan\phi = (1-e^2)\tan\phi = \tan\phi' \qquad \left(e = \sqrt{1-\frac{b^2}{a^2}}\right)$$

THE HORIZONTAL SYSTEM



zenith distance = 90 - a

altitude (elevation) [-90, 90]

azimuth [0, 360]

THE EQUATORIAL SYSTEM