$\beta = \frac{1}{kT}$ $\longrightarrow$ k_B bir sabit, T sıcaklık

$Z = Tr\, e^{-\beta \mathcal{H}}$ bölüşüm fonksiyonu

S_i, S_j occupation number'lar (spinler)

J bir enerji çarpanı

H manyetik alan

M mıknatıslanma

N spin sayısı olmak üzere

$\left(-\beta \mathcal{H} = J \sum_{ij} S_i S_j + H \sum_i S_i = \ln Z \right)$ $\longrightarrow$ açarak yazmadım

$$\langle S_i \rangle = \frac{Tr\, S_i\, e^{-\beta \mathcal{H}}}{Tr\, e^{-\beta \mathcal{H}}} = \frac{1}{N} \frac{\partial \ln Z}{\partial H} = \frac{1}{NZ} \frac{\partial}{\partial H} Tr\, e^{-\beta \mathcal{H}}$$

$$= \frac{1}{NZ} Tr \left(\sum_i^N S_i \right) e^{-\beta \mathcal{H}}$$

$$\chi = \frac{\partial M}{\partial H} = \frac{\partial \langle S_i \rangle}{\partial H} = \frac{\partial}{\partial H} \left(\frac{1}{NZ} Tr \left(\sum_i^N S_i \right) e^{-\beta \mathcal{H}} \right) = \frac{Tr \left(\sum_i S_i \right) e^{-\beta \mathcal{H}} \left(\sum_j S_j \right)}{NZ}$$

↓ diagonallık

$$- \frac{\left[Tr \left(\sum_i S_i \right) e^{-\beta \mathcal{H}} \right] \left[Tr \left(\sum_j S_j \right) e^{-\beta \mathcal{H}} \right]}{NZ^2}$$

χ

$$\frac{1}{N}\sum_{i,j}\left(\frac{\mathrm{Tr}\, s_i s_j e^{-\beta\mathcal{H}}}{Z} - \frac{\mathrm{Tr}\, s_i e^{-\beta\mathcal{H}}}{Z}\,\frac{\mathrm{Tr}\, s_j e^{-\beta\mathcal{H}}}{Z}\right) = \frac{1}{N}\sum_{i,j}\left(\langle s_i s_j\rangle - \langle s_i\rangle\langle s_j\rangle\right)$$

$$= \frac{1}{N}\sum_{ij}\langle (s_i - \langle s_i\rangle)(s_j - \langle s_j\rangle)\rangle = \frac{1}{N}\sum_{ij}\Gamma(i,j) = \sum_{j}\Gamma(i,j) = \chi$$

$\Gamma(i,j)$ → fektetzelitken