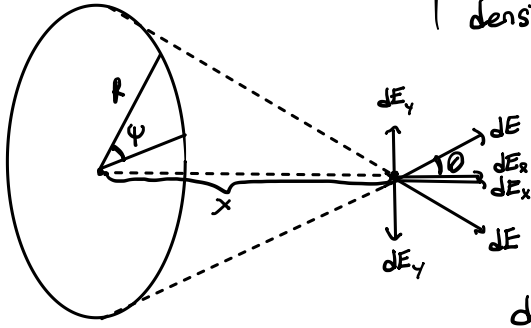


(charge density: λ) $\left(\lambda = \frac{Q}{2\pi R} \right)$



dE_y 's cancel out due to symmetry

$$dE_x = dE \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{dq}{(x^2+R^2)^{3/2}} \frac{x}{\sqrt{x^2+R^2}} \hat{x}$$

$$\left. \begin{aligned} dq &= \lambda ds \\ ds &= R d\psi \end{aligned} \right\} \rightarrow dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda R x}{(x^2+R^2)^{3/2}} \hat{x} d\psi$$

$$\int dE_x = \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda R x}{(x^2+R^2)^{3/2}} \hat{x} d\psi = \frac{1}{4\pi\epsilon_0} \frac{\lambda R x}{(x^2+R^2)^{3/2}} \cdot 2\pi \hat{x} = \frac{R x}{2\epsilon_0 (x^2+R^2)^{3/2}} \cdot \frac{Q}{2\pi R} \hat{x} = \frac{1}{4\pi\epsilon_0} \frac{Q x}{(x^2+R^2)^{3/2}} \hat{x} = \frac{k Q x}{(x^2+R^2)^{3/2}} \hat{x} = \vec{E}$$

