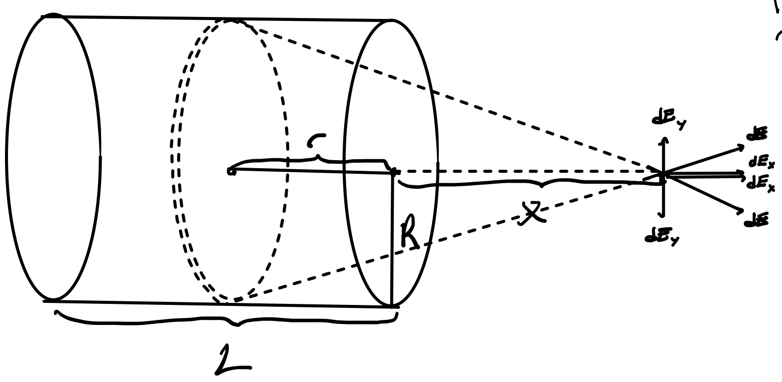
$\left(\text{charge density: } \rho\right) \quad \left(\rho = \dfrac{Q}{\pi R^2 L}\right)$



from the $\vec{E}$ of a uniformly charged circle

$$dE_x = \frac{1}{2\pi\varepsilon_0}\frac{dq}{R^2}\left(1 - \frac{x+r}{\sqrt{(x+r)^2+R^2}}\right)\hat{x}$$

$$dq = \rho\, dV$$

$$dV = \pi R^2 dr$$

$$dE_x = \frac{\rho}{2\varepsilon_0}\left(1 - \frac{x+r}{\sqrt{(x+r)^2+R^2}}\right)\hat{x} \longrightarrow \int dE_x = \int_0^L \frac{\rho}{2\varepsilon_0}\left(1 - \frac{x+r}{\sqrt{(x+r)^2+R^2}}\right)\hat{x}\,dr = \frac{\rho}{2\varepsilon_0}\left(L - \sqrt{(x+L)^2+R^2} + \sqrt{x^2+R^2}\right)\hat{x} = \frac{1}{2\varepsilon_0}\frac{Q}{\pi R^2 L}\left(L - \sqrt{(x+L)^2+R^2} + \sqrt{x^2+R^2}\right)\hat{x}$$

$$\frac{1}{2\pi\varepsilon_0}\frac{Q}{R^2 L}\left(L - \sqrt{(x+L)^2+R^2} + \sqrt{x^2+R^2}\right)\hat{x} = \frac{2kQ}{R^2 L}\left(L - \sqrt{(x+L)^2+R^2} + \sqrt{x^2+R^2}\right)\hat{x} = \vec{E}$$