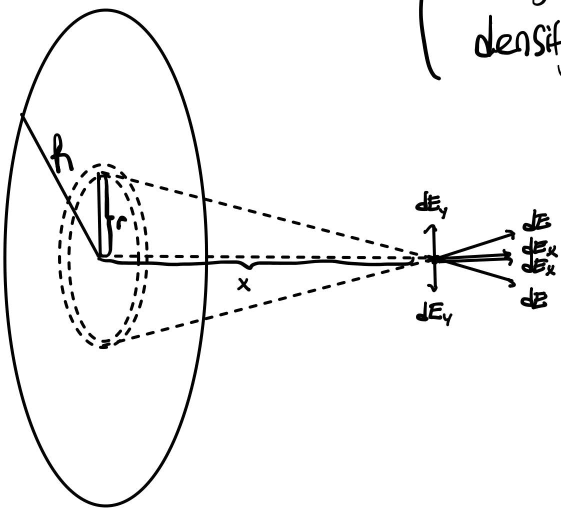
$$\left(\text{charge density} : \sigma\right) \quad \left(\sigma = \frac{Q}{\pi R^2}\right)$$



from the $\vec{E}$ of a uniformly charged hoop

$$\left.\begin{aligned} dE_x &= \frac{1}{4\pi\epsilon_0}\frac{dq \cdot x}{(x^2+r^2)^{\frac{3}{2}}}\,\hat{x} \\ dq &= \sigma\, dA \\ dA &= 2\pi r\, dr \end{aligned}\right\} \rightarrow dE_x = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma x r\, dr}{(x^2+r^2)^{\frac{3}{2}}}\,\hat{x}$$

$$\int dE_x = \int_0^R \frac{1}{2\epsilon_0}\frac{\sigma x r}{(r^2+x^2)^{\frac{3}{2}}}\,\hat{x}\, dr = \frac{1}{2\epsilon_0}\sigma x\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)\hat{x} = \frac{1}{2\epsilon_0}\frac{Q}{\pi R^2}\, x\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)\hat{x} = \frac{1}{2\pi\epsilon_0}\frac{Q}{R^2}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)\hat{x} = \frac{2kQ}{R^2}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)\hat{x} = \vec{E}$$