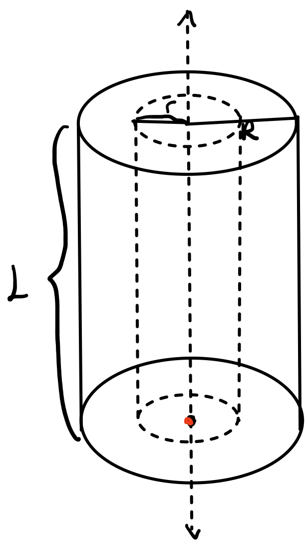
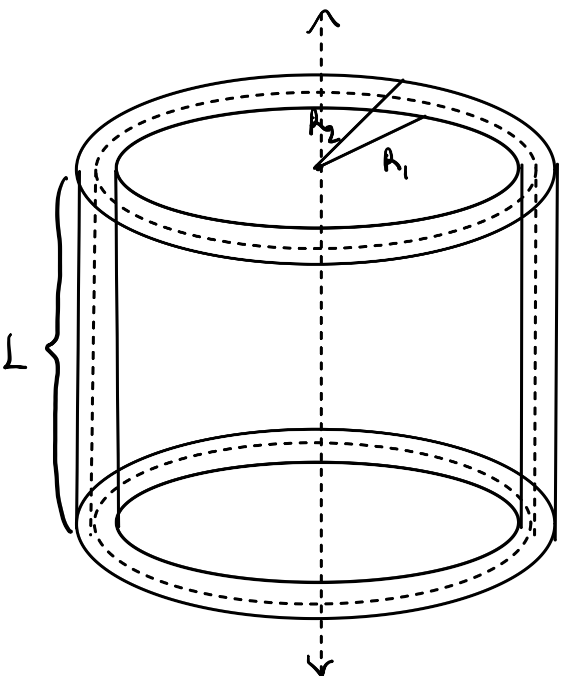


$\begin{pmatrix}\text{mass} \\ \text{density} : \rho\end{pmatrix} \quad \left(\rho = \dfrac{M}{\pi R^2 L}\right)$

$$\left.\begin{aligned} dm &= \rho\, dV \\ dV &= 2\pi L r\, dr \end{aligned}\right\} \rightarrow dm = 2\pi \rho L r\, dr$$

$$I = \int r^2 dm = \int_0^R r^2\, 2\pi \rho L r\, dr = \frac{\pi \rho L R^4}{2} = \frac{\pi L R^4}{2} \cdot \frac{M}{\pi R^2 L} = \frac{1}{2} M R^2$$

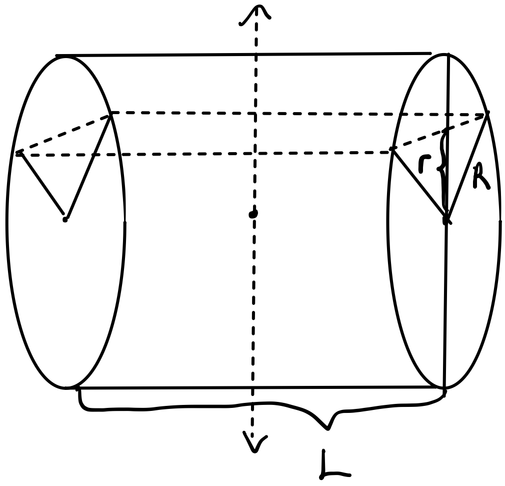


$\begin{pmatrix}\text{mass} \\ \text{density} : \rho\end{pmatrix} \quad \left(\rho = \dfrac{M}{\pi L (R_2^2 - R_1^2)}\right)$

$$\left.\begin{aligned} dm &= \rho\, dV \\ dV &= 2\pi L r\, dr \end{aligned}\right\} \rightarrow dm = 2\pi \rho L r\, dr$$

$$I = \int r^2 dm = \int_{R_1}^{R_2} 2\pi \rho L r^3 dr = \frac{\pi \rho L (R_2^4 - R_1^4)}{2}$$

$$\frac{\pi L (R_2^4 - R_1^4)}{2} \cdot \frac{M}{\pi L (R_2^2 - R_1^2)} = \frac{1}{2} M (R_2^2 + R_1^2)$$



(mass density : ρ) $\left(\rho = \dfrac{M}{\pi R^2 L}\right)$

from the I of an uniform rectangular slab

$$dI = \frac{1}{12} dm \left[L^2 + 4(R^2 - r^2) \right]$$

$$dm = \rho\, dV$$

$$dV = 2L\sqrt{R^2 - r^2}\, dr$$

$$\rightarrow dI = \frac{1}{6} \rho L \sqrt{R^2 - r^2} \left[L^2 + 4(R^2 - r^2) \right] dr$$

$$\int dI = 2 \int_0^R \frac{1}{6} \rho L \sqrt{R^2 - r^2} \left[L^2 + 4(R^2 - r^2) \right] dr = \frac{1}{12} \pi \rho R^2 L^3 + \frac{1}{4} \pi \rho R^4 L = \frac{1}{12} M L^2 + \frac{1}{4} M R^2$$

for each half cylinder