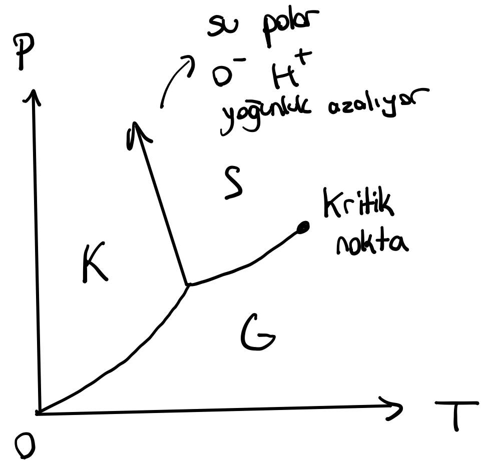
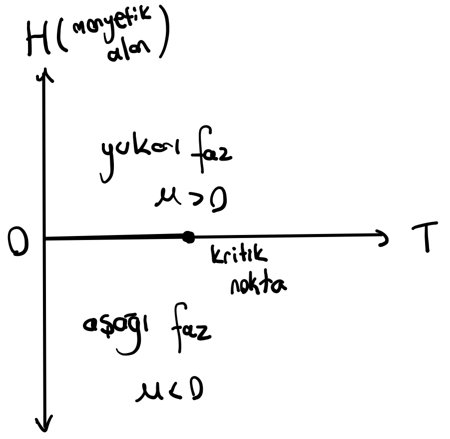


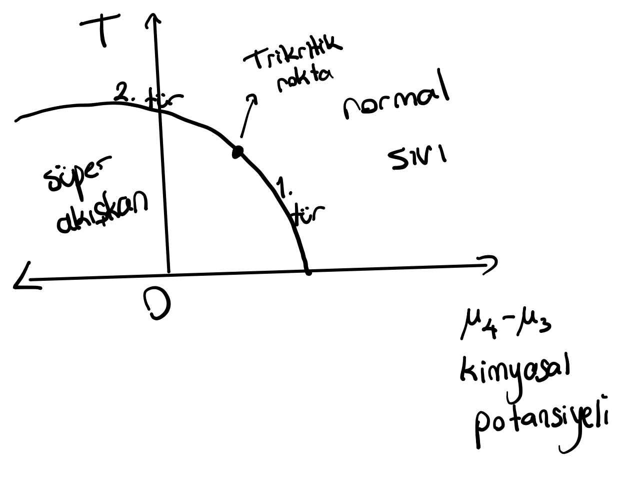


Birinci tür faz geçişi:

- Yoğunlukta/mıknatıslanmada süreksiz değişiklik
- Birden fazla faz bir arada görülür

İkinci tür faz geçişi:

- Sistem tekdüze ancak yoğunluk dalgalanmaları var
- Her ölçekte uzun ömürlü dalgalanmalar.
- Bağlantı uzunluğu (dalgalanmaların ortalama uzunluğu) sonsuz
- Termodinamik fonksiyonlarda tekillikler var.
- Tepki fonksiyonlarında ıraksamalar olabilir



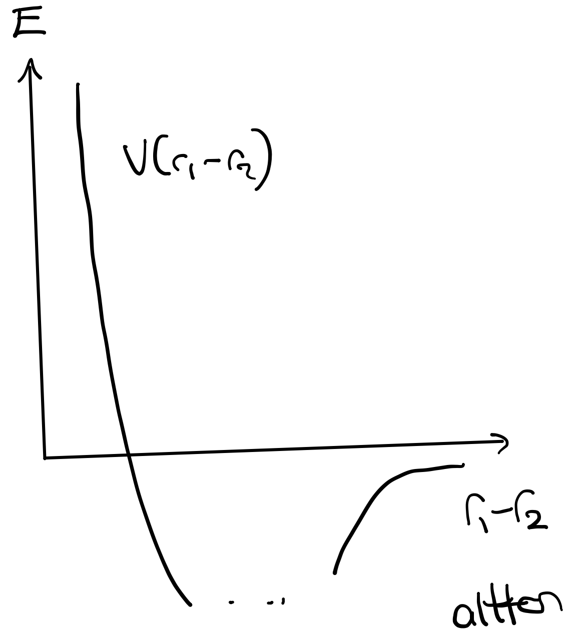
Hemholz serbest enerji $= F = -kT \ln Z = -kT \ln \sum_{\{\vec{r}_i, \vec{s}_i\}} e^{-\overset{E_i}{\mathcal{H}}/kT}$

$$\left.\begin{array}{c} N \to \infty \\ \left(\frac{3}{2} N k_B T = E\right) \\ V \to \infty \\ \frac{N}{V} = \rho \text{ sobit} \end{array}\right\} \begin{array}{c}\text{Termodinamik} \\ \text{Limit}\end{array}$$

Termodinamik limit olması için:

Gazda: $V(r_1 - r_2 \to \infty) \longrightarrow 0$

$V(r_1 - r_2 \to 0) \longrightarrow \infty$



altten da üstten de dur

Kritik olgular

(A) Fiziksel mesafe ölçeğinin yok olması

$\rho_r - \langle \rho_r \rangle =$ ortalamadan sapma

$$\langle (\rho_r - \langle \rho_r \rangle)(\rho_{r'} - \langle \rho_{r'} \rangle) \rangle \sim \frac{e^{-|\vec{r} - \vec{r}'|/\xi}}{|\vec{r} - \vec{r}'|^{u}}$$

Bağlantı mesafesi

$\longrightarrow \infty$

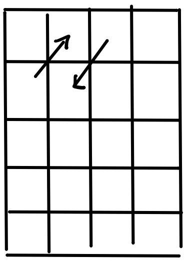
$$\| $$

$$\langle \rho_r \rho_{r'} \rangle - \langle \rho_r \rangle \langle \rho_{r'} \rangle$$

Ising Modeli :

$$-\frac{E}{kT} = -\beta E = J\sum_{\langle ij\rangle} s_i s_j + H\sum_i s_i \quad , \quad Z = \sum_{\{s\}}^{2^N} e^{-\beta E} = \mathrm{Tr}\, e^{-\beta E}$$

$s_i = \pm 1$



$$M = \langle s_i \rangle = \frac{\mathrm{Tr}\, s_i e^{-\beta E}}{\mathrm{Tr}\, e^{-\beta E}} = \frac{1}{N}\frac{\partial \ln Z}{\partial H}$$

$$\frac{1}{N}\frac{\partial \ln Z}{\partial H} = \frac{1}{NZ}\frac{\partial Z}{\partial H} = \frac{1}{NZ}\frac{\partial}{\partial H}\mathrm{Tr}\, e^{J\sum_{\langle ij\rangle} s_i s_j + H\sum_i^N s_i} = \frac{1}{NZ}\mathrm{Tr}\left(\sum_i^N s_i\right)e^{-\beta E} = \langle s_i\rangle \frac{1}{Z}\mathrm{Tr}\, e^{-\beta E}$$

$$= M\frac{1}{Z}\cdot Z = M$$

$$\chi = \frac{\partial M}{\partial H} = \frac{\partial}{\partial H}\left(\frac{1}{NZ}\mathrm{Tr}\left(\sum_i^N s_i\right)e^{-\beta E}\right) = \frac{\mathrm{Tr}\left(\sum_i s_i\right)e^{-\beta E}\left(\sum_j s_j\right)}{NZ}$$

$\downarrow$

alınganlık

$$- \frac{\left[\mathrm{Tr}\left(\sum_i s_i\right)e^{-\beta E}\right]\left[\mathrm{Tr}\left(\sum_j s_j\right)e^{-\beta E}\right]}{NZ^2}$$

$$\frac{1}{N}\sum_{i,j}\left(\frac{\mathrm{Tr}\, s_i s_j e^{-\beta E}}{Z} - \frac{\mathrm{Tr}\, s_i e^{-\beta E}}{Z}\,\frac{\mathrm{Tr}\, s_j e^{-\beta E}}{Z}\right) = \frac{1}{N}\sum_{i,j}\left(\langle s_i s_j\rangle - \langle s_i\rangle\langle s_j\rangle\right)$$

$$= \frac{1}{N}\sum_{i,j}\langle (s_i - \langle s_i\rangle)(s_j - \langle s_j\rangle)\rangle = \frac{1}{N}\sum_{i,j}\Gamma(i,j)$$

$$\chi = \sum_{\vec{r}} \Gamma(\vec{r}) \overset{\text{Fourier}}{=} \Gamma(\vec{k}=0) \sim \int_0^L \Gamma(r)\, r^{d-1} dr$$

Kritik noktada:

$$\chi \sim \int_0^L r^{d-1} \cdot \frac{1}{r^{d-2+\eta}} \cdot dr = \int_0^L r^{1-\eta} dr = L^{2-\eta} \int_0^1 x^{1-\eta} dx \to \infty$$

$$\frac{L^{2-\eta}}{2-\eta}$$

$$\left(x \equiv \frac{r}{L}\right)$$

Termodinamik limitte $L \to \infty$

Tipik olarak $0 \lesssim \eta \lesssim \frac{1}{4}$

Kritik Nokta Yakınında:

$$\chi \sim \int_0^L r^{1-\eta} e^{\frac{-r}{\xi}} dr \sim \int_0^{\xi} r^{1-\eta} dr = \xi^{2-\eta}$$

ξ'dan sonra 0

Eğer bağlantı mesafesi $\xi \sim |T-T_c|^{-\nu} \to \infty$

alınganlık $\chi \sim |T-T_c|^{-\gamma}$, $\gamma = (2-\eta)\nu$

ν, γ kritik üsteller

$$-4 \leq \Gamma(r) = \langle (s_i - \langle s_i \rangle)(s_j - \langle s_j \rangle) \rangle \leq 4$$

(yavaş gidiyor)

$\sum_r \Gamma(r) \to \infty$ ancak termodinamik limitten ve $\Gamma(r)$'deki uzak mesafelerin katkısından gelebilir.

Önemli faktörler:

- Uzayın boyutu
- Serbesti derecesinin simetrisi
- Uzun menzil

Alınganlık $\chi = \frac{\partial M}{\partial H} \sim |T - T_c|^{-\gamma} \rightarrow \infty$

Özgül ısı $C = \frac{\partial U}{\partial T} \sim |T - T_c|^{-\alpha} \rightarrow \infty$

Mıknatıslanma $M \sim (T_c - T)^{\beta} \rightarrow 0$

Bağlantı mesafesi $\xi \sim |T - T_c|^{-\nu} \rightarrow \infty$

Kritik noktada termodinamik tekillikler

Evrensellik

Çok değişik sistemler için kritik üsteller tamamen aynı

Ferromagnet	$YFeO_3$	Üsteller aynı
Akışkan	CO_2, Xe	
Antiferromagnet	FeF_2	
Alaşım	β-brass	
Moleküler kristal	NH_4Cl	
Herhangi bir kristal yapısı	$d=3$ Ising	

Süperakışkan Helium	Evrensellik sınıfı
$\hat{\mathrm{①}}$ mayetik sistem	
sıvı kristaller	

Teori tarihi:

Ortalama alan teorisi (1960) $\rightarrow$ Curie-Weiss, van der Waals, Hartree, Landau, Gibbs

Yüksek dereceli pertürbasyon teorisi

Monte Carlo hesapları

Moleküler dinamik

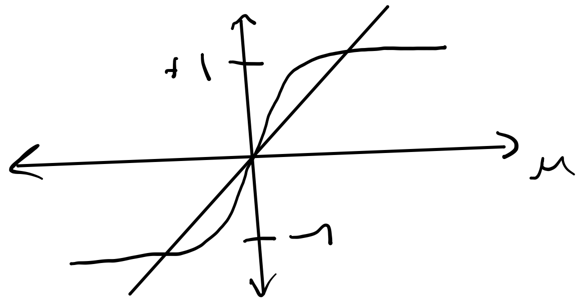
Kadanoff ölçekleme teorisi (1966)

K.G. Wilson Renormalizasyon Grubu Teorisi (1970)

1) Basit Ortalama alan

komşu sayısı

Tek spin enerjisi : $-\beta E_1 = s_1(qJ\mu + H)$



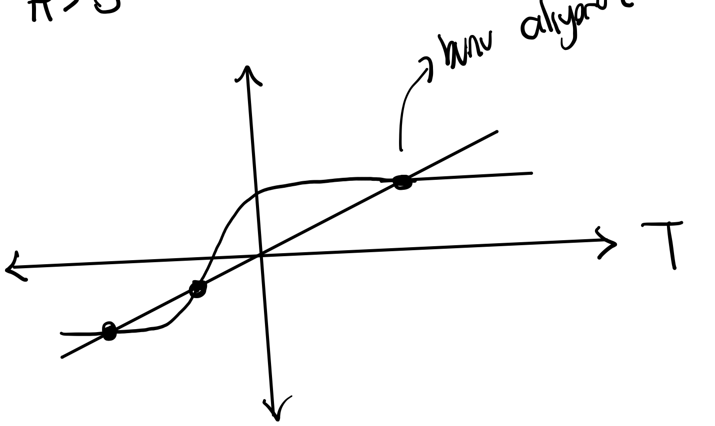
$$\mu = \langle s_1 \rangle = \frac{\sum_{s_1} s_1 e^{-\beta E_1}}{\sum_{s_1} e^{-\beta E_1}} = \tanh(qJ\mu + H)$$

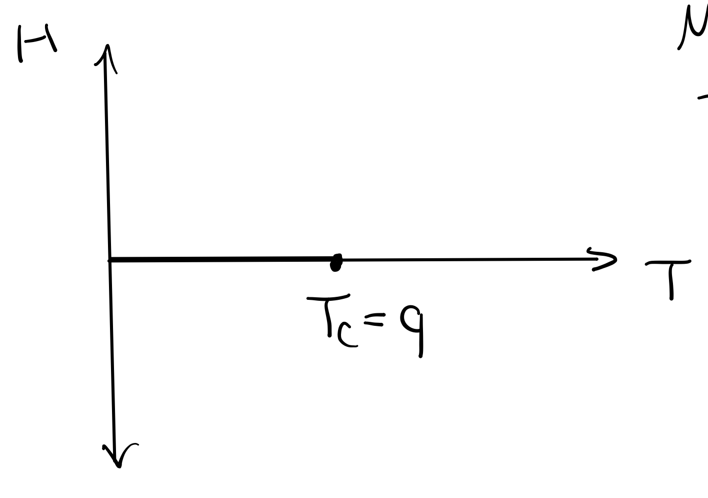
$$1 = \frac{d}{d\mu} \tanh(qJ\mu)\Big|_{\mu=0} = qJ_c \, , \; T_c = \frac{1}{J_c} = q$$

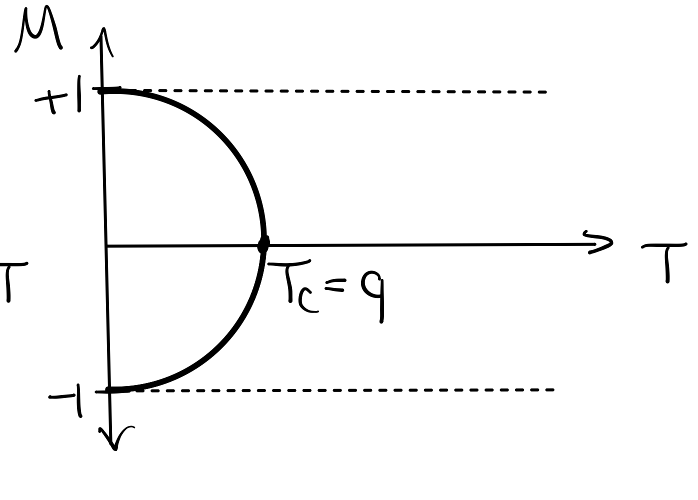
Helmholz's free energy

$$F = -kT\ln Z = -kT\ln \sum_{s_1}^{\mp 1} e^{s_1(qJ\mu + H)} = -kT\ln\left(2\cosh(qJ\mu + H)\right)$$

$H > 0$







Ortalama alan hangi şartlarda daha geçerli?

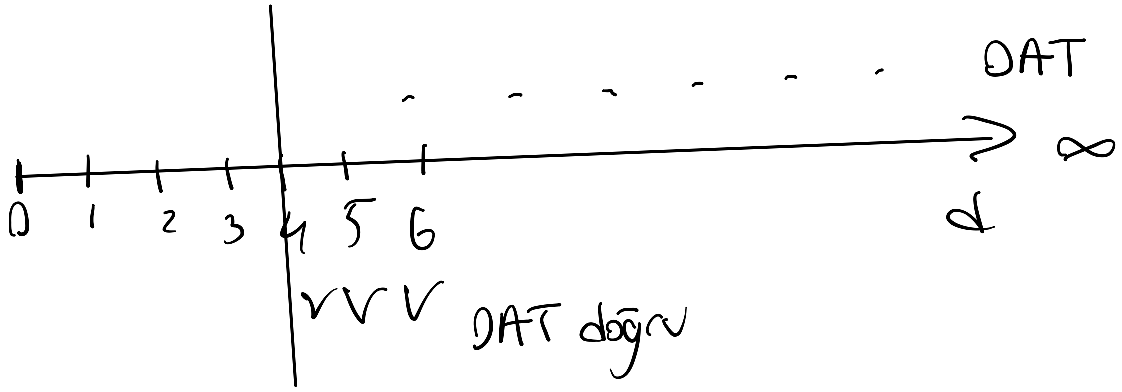
eşdeğer komşu modeli

$$\sum_j s_j = NM = qM$$

$$J\sum_j s_i s_j = J s_i q M$$

uzun menzilli etkileşmeler

Çok boyutlar



Gibbs Varyasyonel Prensibi

$$F = -kT\ln Z \le \mathrm{Tr}\,\rho\mathcal{H} + kT\,\mathrm{Tr}\,\rho\ln\rho \quad , \quad \mathrm{Tr}\,\rho = 1$$

eşitlik olunca $\rho = \dfrac{e^{-\beta\mathcal{H}}}{\mathrm{Tr}\,e^{-\beta\mathcal{H}}} = \rho_{canonical}$

$$\mathrm{Tr}\,\rho_{can}\mathcal{H} = \langle\mathcal{H}\rangle = U$$

$$-k\,\underbrace{\mathrm{Tr}\,\rho_{can}\log\rho_{can}}_{\text{entropi}} = S$$

$$\Psi(\rho) = Tr\rho\mathcal{H} + kT\,Tr\,\rho\ln\rho$$

$$\Psi(\rho_{can}) = U - TS = F$$

$$\Psi(\rho) = \sum_n \rho_n\left(\langle n\mathcal{H}n\rangle + kT\ln\rho_n\right) \rightarrow \text{minimum olsun}$$

Lagrange multiplier

sabit

$$\frac{\partial}{\partial\rho_n}\left(\Psi + \Lambda Tr\rho\right) = \langle n\mathcal{H}n\rangle + kT\ln\rho_n^* + kT + \Lambda$$

$$-\frac{\langle n\mathcal{H}n\rangle}{kT} + \text{sabit} = \ln\rho_n^*$$

$\sum_i p_i = 1$ olacak şekilde seçtik

$$e^{-\frac{\langle n\mathcal{H}n\rangle}{kT} + \text{sabit}} = \rho_n^* = \frac{e^{-\beta\langle n\mathcal{H}n\rangle}}{\sum_{n'} e^{-\beta\langle n'\mathcal{H}n'\rangle}}$$

ikinci türev $= \frac{kT}{\rho_n} \rightarrow$ her zaman pozitif

Tek nokta faktörize : $\prod_{i=1}^{N} \rho_i = \rho$

Çift nokta faktörize : $\prod_{ij} \rho_i\rho_j = \rho$ Bethe-Peierls

---

$$\ln\rho_i = a + Bs_i$$

$$\rho_i = e^a + e^{Bs_i} = Ae^{Bs_i}$$

$$\sum_{s_i} \rho_i = 2A\cosh(B) = 1 \Rightarrow A = \frac{1}{2\cosh(B)} \quad \text{normalize}$$

$$\left(Tr = \sum_{\{s_i\}}^{2^N} = \prod_i^N \sum_{s_i}^{\pm 1}\right)$$

$$-\beta Tr\rho\mathcal{H} = Tr\prod_{i'}\rho_{i'}\left(J\sum_{ij}s_is_j + H\sum_i s_i\right) = J\sum_{ij}(tr_i\,\rho_is_i)(tr_j\,\rho_js_j) + H\sum_i(tr_i\,\rho_is_i)$$

$$J\sum_{ij} m^2 + H\sum_i m = \frac{JNq}{2}m^2 + HNm$$

$$m = \mathrm{tr}_i\, p_i s_i = \sum_{s_i}^{\pm 1} A e^{Bs_i} s_i = 2A\sinh(B) = \tanh(B)$$

$$\mathrm{Tr}\, p\ln p = \mathrm{Tr}\prod_i p_i \left(\sum_j \ln p_j\right) = \sum_j \mathrm{tr}_j\, p_j \ln p_j = N\, \mathrm{tr}_j\, p_j \left(B\underset{a}{s_i} + \ln A\right) = N(Bm + \ln A)$$

$$-\frac{\beta\Psi}{N} = \frac{Jqm^2}{2} + Hm - Bm - \ln A$$

$$\cosh^2(B) = (2A)^{-2}, \quad \sinh^2(B) = (2A)^{-2} - 1, \quad m^2 = \tanh^2(B) = 1 - 4A^2$$

$$A = \frac{1}{2}(1-m^2)^{\frac{1}{2}}, \quad B = \tanh^{-1}(m) = \frac{1}{2}\ln\left(\frac{1+m}{1-m}\right)$$

$$-\frac{\beta\Psi}{N} = \frac{Jqm^2}{2} + Hm - m\tanh^{-1}(m) - \frac{1}{2}\ln(1-m^2) + \ln 2$$

$$= \frac{Jqm^2}{2} + Hm - \frac{(1+m)}{2}\ln(1+m) - \frac{(1-m)}{2}\ln(1-m)$$

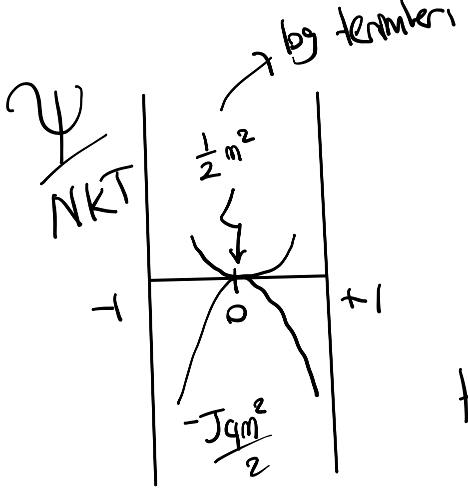
minimize etmek için

$$0 = \frac{d}{dm}\left(-\frac{\beta\Psi}{N}\right) = Jqm + H - \mathrm{arctanh}(m) \Rightarrow m = \tanh(Jqm + H)$$

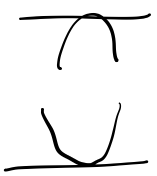
$\Downarrow$

Özfutarlılık şartı

Bragg-Williams Şekli



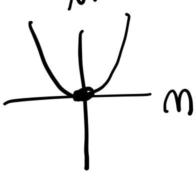
$$\frac{1}{2}(1-Jq)m^2 \Longrightarrow \begin{array}{l} J > q^{-1} \\ J < q^{-1} \end{array}$$



$H=0$

$J < J_c = q^{-1}$

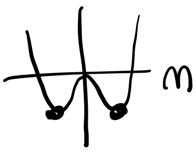
$T > T_c = q$



$H=0$

$J > J_c = q^{-1}$

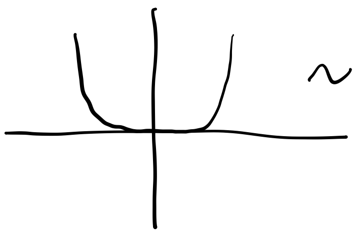
$T < T_c = q$



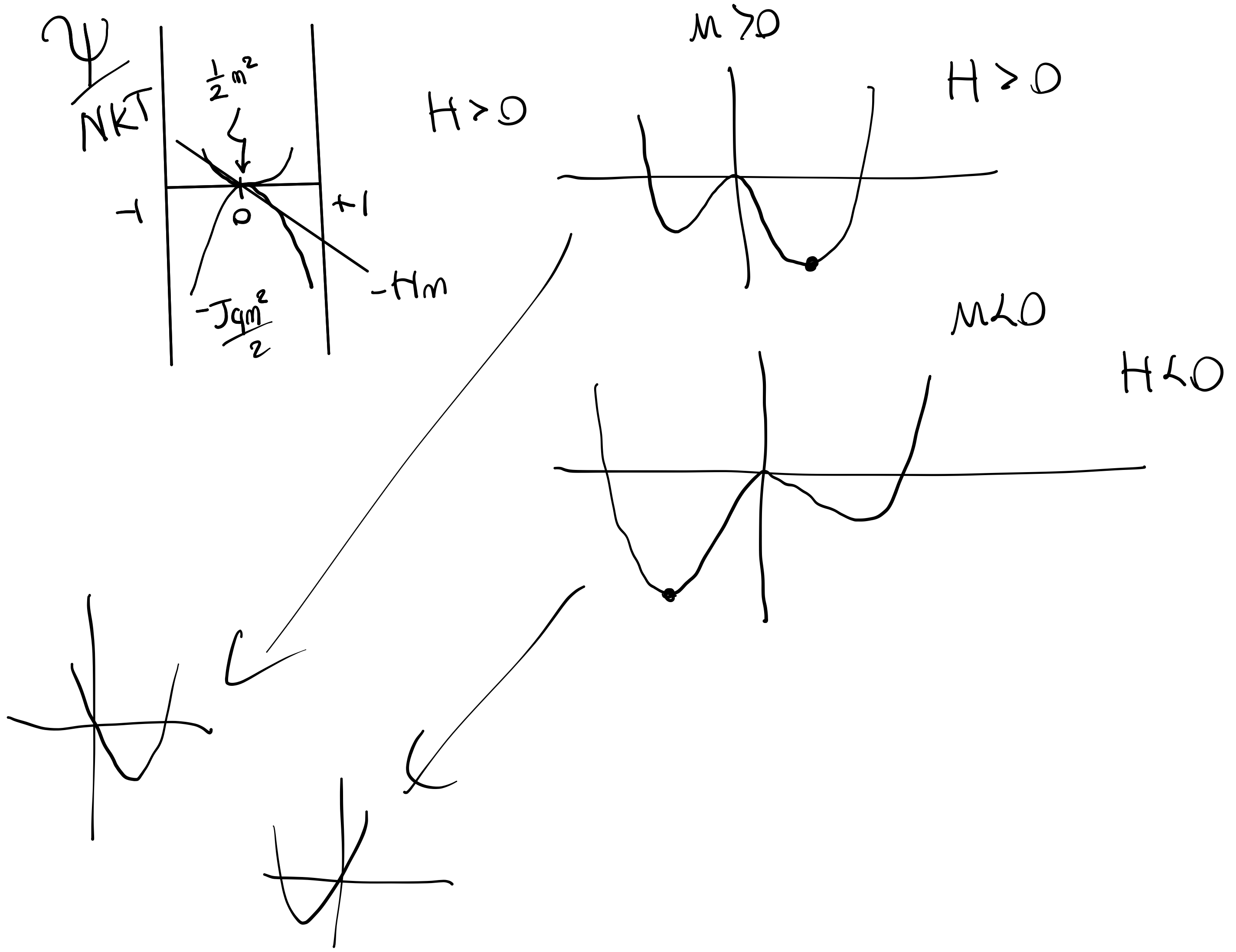
$H=0$

$J = J_c$

$T = T_c$



$\sim m^4 \quad H=0$



$$\frac{\psi}{NKT} = -\frac{Jqm^2}{2} - Hm + \frac{(1+m)}{2}\ln(1+m) + \frac{(1-m)}{2}\ln(1-m)$$

$$\left.\frac{d\psi}{dm}\right|_M = 0$$

Fiziksel nokta

$M = M(J,H)$

ikinci türde ikisi bir arada

birinci türde

$\psi(M_1) = \psi(M_2)$

Kritik nokta

$M = M(J,H)$, $\dfrac{d^2\psi}{dm^2} = 0$ ve $\dfrac{d^3\psi}{dm^3} = 0$

Birinci tür faz geçişi $\Psi(M_1) = \Psi(M_2)$

Landau

Serbest enerji fonksiyonu

$\Psi(m; H, T, L)$

var ki Ψ (m'ye göre minimize edilmiş) $\simeq F$

m (minimize eden) $\simeq M$

Ψ'nin m'ye göre açılımı var

$\frac{\partial \Psi}{\partial M} = 0$ and $\frac{\partial^2 \Psi}{\partial M^2} > 0$ $\longrightarrow$ durum denklemi

$\frac{\partial^3 \Psi}{\partial M^3} = 0$ and $\frac{\partial^4 \Psi}{\partial M^4} > 0$ $\longrightarrow$ kritik nokta

ikinci tür

$\left.\frac{\partial \Psi}{\partial M}\right|_{m_1, m_2} = 0$, $\Psi(m_1) = \Psi(m_2) \longrightarrow$ birinci tür

Kritik nokta $(T = T_c ; H = H_c = 0, L = L_c = 0, M = M_c = 0)$

$$\Psi = \Psi_c + a_0 + a_1 m + a_2 m^2 + a_3 m^3 + a_4 m^4$$

$$a_n = a_{n1}(H - H_c) + a_{n2}(T - T_c) + a_{n3}(L - L_c)$$

$a_4 = \underset{\text{pozitif}}{C} \ldots$

Simetri : $\Psi(m; H, T, L) = \Psi(-m; -H, T, -L)$

$$a_{13} = a_{21} = a_{23} = a_{32} = 0$$

$$\left.\frac{\partial \psi}{\partial m}\right)_M = 0 = a_1 + 2a_2 M + 3a_3 M^2 + 4cM^3$$

$$\frac{\partial^2 \psi}{\partial m^2} = 0 = 2a_2 + 6a_3 M + 12cM^2$$

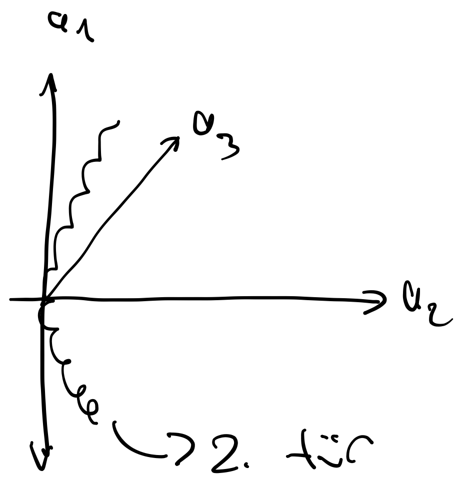
$$\frac{\partial^3 \psi}{\partial m^3} = 0 = 6a_3 + 24cM$$

$$\Bigg\} \quad 0 = 2a_2 - 12cM^2$$

$$M_c = -\frac{a_3}{4c}$$

$$a_2 = \frac{3}{8}\frac{a_3^2}{c}$$

$$a_1 = \frac{a_3^3}{16c^2}$$



$M=0$'da $a_2 - a_1$ düzlemi 1. tür

$$\psi(m) = \psi_c + a_0 + a_1 M + a_2 M^2 + a_4 M^4$$

$a_3 = 0$

$$= \psi_c + a_0 - a_2 M^2 - 3a_4 M^4$$

$\psi(M_1) = \psi(M_2)$ arıyoruz

$a_1 = 0$ için buluyoruz : $0 = (a_2 + 4cM^2)\, M \to 0$

$M = 0$ hep çözüm $\qquad a_2 < 0$ için $\quad M = \pm\sqrt{\frac{-a_2}{2c}}$

minimize

a_3 = etkisiz alan
a_2 = zayıf alan
a_1 = kuvvetli

alan

Durum denklemi

$$\left.\frac{\partial \psi}{\partial m}\right)_M = 0 = a_1 + 2a_2 M + 3a_3 M^2 + 4cM^3$$

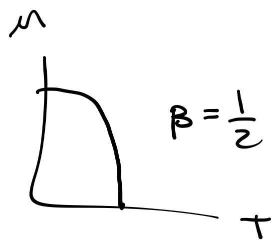
Mıknatıslanma M : (A) Sıcaklıktan

$a_1 = 0, \quad a_3 = 0$

$2a_2 M + 4cM^3 = 0$

$a_2 > 0 \Rightarrow M = 0$

$a_2 < 0 \Rightarrow M \pm \sqrt{\frac{-a_2}{2c}} \sim \left(\frac{T_c - T}{T_c}\right)^{\frac{1}{2}}$



$\beta = \frac{1}{2}$

(B) Manyetik alandan

Simetri bozan yönde

$a_2 = 0 \quad a_3 = 0$

$a_1 + 4cM^3 = 0$

$$M = \left(\frac{-a_1}{4c}\right)^{\frac{1}{3}} \sim H^{\frac{1}{3}} \text{ ve } H \sim M^{\delta} \Rightarrow \delta = 3$$

Alınganlık

$$\chi = \left.\frac{\partial M}{\partial H}\right)_{H=0} \sim -\left.\frac{\partial M}{\partial a_1}\right)_{a_1=0}$$

$$\left.\frac{\partial \psi}{\partial m}\right)_M = 0 = a_1 + 2a_2 M + 3a_3 M^2 + 4cM^3 \quad a_3 = 0$$

$$0 = -1 + (2a_1 + 12cM)\frac{\partial M}{\partial a_1} \Rightarrow \chi = \frac{1}{2a_2 + 12cM^2}$$

$a_2 \to 0^+ : \chi = \frac{1}{2a_2} \to \infty \Rightarrow \chi \sim \left[\frac{T-T_c}{T_c}\right]^{-1}$ $\qquad \gamma = 1$

$a_2 \to 0^- : \chi = \frac{1}{4|a_2|} \to -\infty \Rightarrow \gamma = 1$

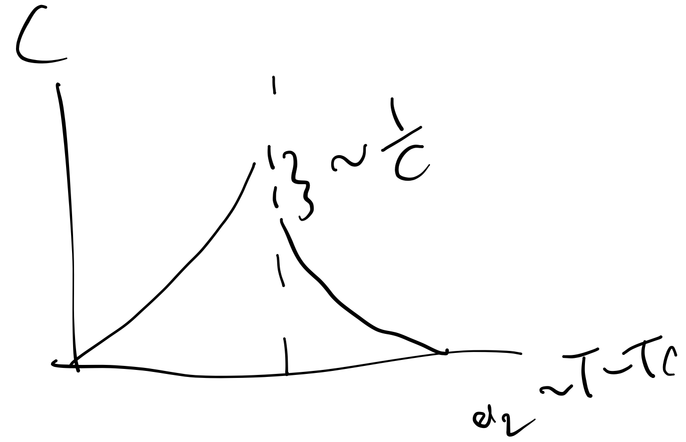
Özgül Isı

$a_1 = 0 \quad a_3 = 0$

$$\frac{F}{N} = \Psi(M) = \Psi_c + a_0 + a_2 M^2 + c M^4$$

$$S = -\frac{1}{N}\frac{\partial F}{\partial T}, \quad C = T\frac{\partial S}{\partial T} = -\frac{T}{N}\frac{\partial^2 F}{\partial T^2}$$

$$\sim \frac{\partial^2 \Psi}{\partial a_2^2}\left[a_{22}\right]^2$$



$$0 = a_1 + 2a_2 M + 4cM^3$$

$$\frac{1}{a_2^{3/2}} : 0 = \frac{a_1}{a_2^{3/2}} + 2\left(\frac{M}{a_2^{1/2}}\right) + 4c\left(\frac{M}{a_2^{1/3}}\right)^3$$

$\to M(a_1, a_2)\, a_2^{\beta}\, \text{fonk}\left(\frac{a_1}{a_2^{\Delta}}\right)$

$$M = a_2^{1/2}\, \text{fonk}\left(\frac{a_1}{a_2^{3/2}}\right)$$

$$M(a_1, a_2) = a_2^{\beta}\, \text{fonk}\left(\frac{a_1}{a_2^{\Delta}}\right)$$

$a_1 = 0 \Rightarrow M = a_2^{\beta} f(0)$

$a_2 = 0 \Rightarrow \text{fonk}(x) = x^4 g(x^{-1})$

$x \to \infty$

$$M = a_2^{\beta - \Delta \upsilon} a_1^{\upsilon} g\left(\frac{a_2^{\Delta}}{a_1}\right) = a_1^{\frac{\beta}{\Delta}} g(0) = a_1^{\frac{1}{\delta}} g(0) \Rightarrow \delta = \frac{\Delta}{\beta} = 3$$

$$F = \Psi_c + a_0 + F_s$$

$$F_s = a_1 M + a_2 M^2 + c M^4$$

$$= a_2^2 \left\{ \frac{a_1}{a_2^{\frac{3}{2}}} \,\text{fonk}\left(\frac{a_1}{a_2^{\frac{3}{2}}}\right) + \text{fonk}\left(\frac{a_1}{a_2^{\frac{3}{2}}}\right)^2 + \text{fonk}\left(\frac{a_1}{a_2^{\frac{3}{2}}}\right)^4 \right.$$

$$= a_2^2 f\left(\frac{a_1}{a_2^{\frac{3}{2}}}\right) = a_2^{2-\alpha} f\left(\frac{a_1}{a_2^{\Delta}}\right) = F_s$$

$$M \sim \frac{\partial F}{\partial a_1} = a_2^{2-\alpha-\Delta} f'(0) \Rightarrow \beta = 2-\alpha-\Delta$$

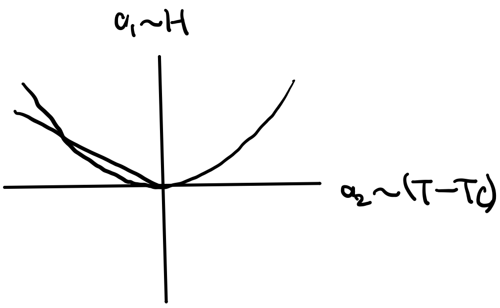
$$\chi \sim \left.\frac{\partial F}{\partial a_1^2}\right|_{a_1=0} \sim a_2^{2-\alpha-2\Delta} \Rightarrow -\gamma = 2-\alpha-2\Delta$$

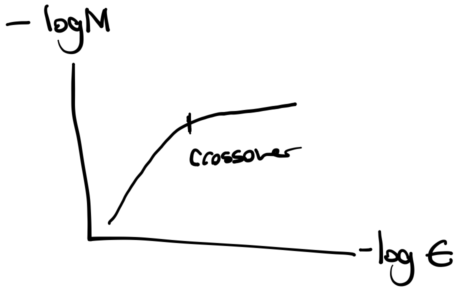
sıcaklık yönü $x \to 0$ manyetik alan yönü $x \to \infty$

$M \sim a_2^{\beta} \,\text{fonk}\left(\frac{a_1}{a_2^{\beta\delta}}\right) \to a_2^{\beta}$ $\to a_1^{1/\delta}$

limit (yakınlık için) $= \frac{a_1}{a_2^{\Delta}} = x = 17$

$a_1 = a_2^{\Delta}\, 17$





$$-\beta\mathcal{H} = J\sum_{\langle ij\rangle} S_iS_j + H\sum_i S_i \qquad S_i = \pm 1$$

d=1 sistemler için olgusal argümanlar

$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}} \sim \omega_m e^{-\beta E_{min}} = e^{-\beta(E_{min} - kT\ln\overset{\text{min sayısı}}{\omega_m})}$$

$$\uparrow\uparrow\ \uparrow\ \uparrow \Rightarrow E_m - kT\ln\omega_m = -N\hat{J}$$

$$\uparrow\uparrow\ \downarrow\ \uparrow \Rightarrow -N\hat{J} + 2\hat{J} - kT\ln N$$

$$\Delta = 2\hat{J} - kTN$$

Transfer matrixi:

$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}} = \sum_{\{s\}} e^{\left(\frac{H}{2}s_1 + Js_1s_2 + \frac{H}{2}s_2\right)\cdots\left(\frac{H}{2}s_N + Js_Ns_1 + \frac{H}{2}s_1\right)}$$

$$e^{\frac{H}{2}s_i + Js_is_j + \frac{H}{2}s_j} = \begin{array}{cc} \begin{bmatrix} e^{J+H} & e^{-J} \\ e^{-J} & e^{J-H} \end{bmatrix} & \begin{array}{l} s_i = 1 \\ s_i = -1 \end{array} \\ \begin{array}{cc} s_j = 1 & s_j = -1 \end{array} & \end{array} = \langle s_i|T|s_j\rangle$$

$$Z = \sum_{\{s\}} \langle s_1|T|s_2\rangle\langle s_2|T|s_3\rangle \cdots \langle s_N|T|s_1\rangle = \sum_{s_1} \langle s_1|T^N|s_N\rangle = \mathrm{Tr}\, T^N$$

T simetrik $\rightarrow$ ortagonal dönüşümle diyagonal olur.

$$S^{-1}TS = \tilde{T} = \begin{bmatrix} \lambda_> & 0 \\ 0 & \lambda_< \end{bmatrix}$$

$$Z = \mathrm{Tr}\,\tilde{T}^N = \mathrm{Tr}\begin{bmatrix} \lambda_>^N & 0 \\ 0 & \lambda_<^N \end{bmatrix} = \lambda_>^N + \lambda_<^N = \lambda_>^N\left(1 + \left(\frac{\lambda_<}{\lambda_>}\right)^N\right)$$

$$f = -\frac{kT}{N}\ln Z = -kT\lambda_> - \frac{kT}{N}\left(\frac{\lambda_<}{\lambda_>}\right)^N$$

$$-kT\ln\left(1 + \left(\frac{\lambda_<}{\lambda_>}\right)^N\right)$$

$x \equiv e^{J} \quad y \equiv e^{H}$

$$\begin{vmatrix} xy-\lambda & x^{-1} \\ x^{-1} & xy^{-1}-\lambda \end{vmatrix} = (xy-\lambda)(xy^{-1}-\lambda) - x^{-2} = 0$$

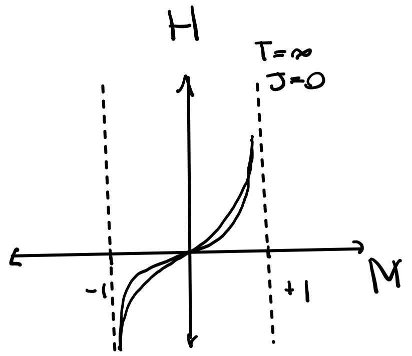
$$\lambda_{\gtrless} - \frac{1}{2}\left[(y+y^{-1})x \pm \left[x^{2}(y+y^{-1})^{2} - 4x^{2} - 4x^{-2}\right]^{\frac{1}{2}}\right] \quad \text{hep pozitif}$$

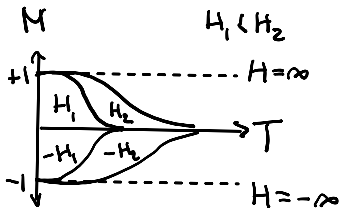
$N \to \infty$ Termo limit

$$f = -\frac{kT}{N} \ln \lambda_{>}$$

$\lambda_{>}$ analitik $J < \infty$ için

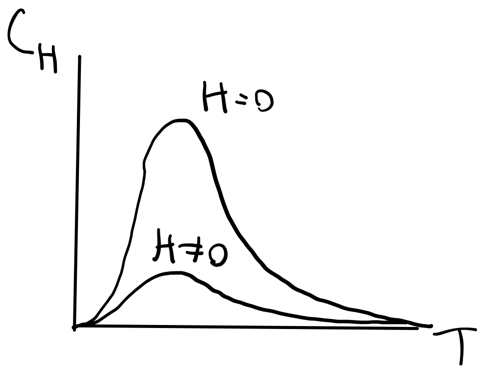
$$M = \langle s_i \rangle = \frac{1}{N} \frac{\partial \ln Z}{\partial H} = \frac{\partial}{\partial H} \ln \lambda_{>}$$

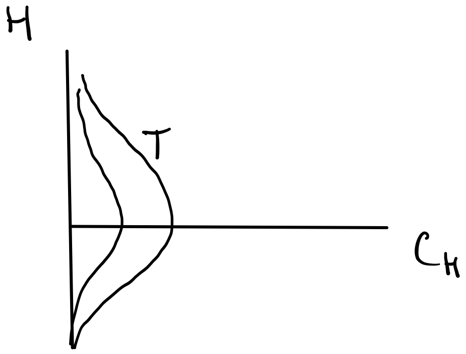




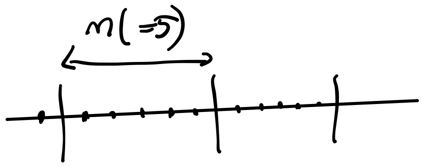
$$U = -\tilde{J} \langle s_i s_j \rangle = \frac{-\tilde{J}}{N} \frac{\partial}{\partial J} \ln Z = -\tilde{J} \frac{\partial}{\partial J} \ln \lambda_{>}$$

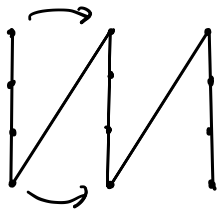
$$C_H = \left.\frac{\partial U}{\partial T}\right|_H$$





Frobenius





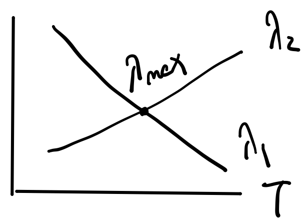
Transfer matrixi $q^m \times q^m$

$$Z = \sum_{n=1}^{q^m} (\lambda_n)^{\frac{N}{m}} \qquad f = \lim_{N\to\infty}\left[\frac{kT}{N}\ln Z\right] = -\frac{kT}{m}\ln\lambda_{max}$$

f nasıl tekil olabilir?

1) $\lambda_{max}(T,H)$ analitik olmayabilir

2) Öz vektörler birbirlerini geçebilir.



(1) Sonlu boyutlu ($q^n<\infty$) ve pozitif elemanlı ($T_{ij}>0$) özdeğerleri dejeneret olamaz.

(2) λ_{max} matriks elemanlarının analitik fonksiyonudur

$$-\beta\mathcal{H} = \sum_{i,r} J(s_i,s_{i+r}) = \sum_{i,r}\frac{\tilde{J}}{kT}(s_i s_{i+r})$$

$$\tilde{J}(r)\sim \frac{1}{r^{2+\epsilon}}$$

1) $\sum_{r=1}^{\infty} |\tilde{J}|_r < \infty$ → faz geçişi yok

2) $M\neq 0$ bazı $T>0$ ve $H=0$ için → faz geçişi var

$$\sum_{r=1}^{\infty}\frac{1}{r^3\tilde{J}} < \infty, \quad \tilde{J} \geq 0$$

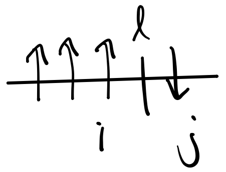
Örnek $\tilde{J}(r)\sim\frac{1}{r^{2-\epsilon}}$

$$Z = \sum e^{-\beta(E_0 - kTw_0)} = e^{-\beta\Psi_0}$$

$$\Psi_0 = E_0 - TS_0 \ , \quad S_0 = k\ln w_0$$

$M_0 = \langle s \rangle_0 > 0$ olur mu?

l'nin sağındakileri çevirelim



$$\Delta E_0 = \sum_{i\le l}\sum_{j>l} 2\tilde{J}(i-j)M_0^2 = 2M_0^2\sum_{r=1}^{\infty} r\,J(r)$$

$$\Delta S_0 = k\log N$$

$$\Delta\Psi_0 = 2M_0^2\sum_{r=1}^{N} r\tilde{J}(r) - kT\ln N \overset{?}{\gtrless} 0$$

1) $\sum_{r=1}^{N} r\tilde{J}(r) < \ln N \Longrightarrow M_0 = 0$

D. Ruelle $\left(\tilde{J} \sim \frac{1}{r^{2+\epsilon}}\right)$

2) $\sum_{r=1}^{N} r\tilde{J}(r) > \ln N \Longrightarrow M_0 > 0$ tutarlı

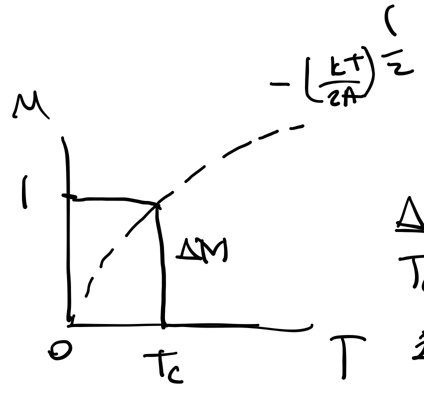
F. Dyson $\left(\tilde{J} \sim \frac{1}{r^{2-\epsilon}}\right)$

3) $\sum_{r=1}^{N\to\infty} r\tilde{J}(r) \simeq A\ln N$

$$\Delta\Psi_0 = (2M_0^2A - kT)\ln N = \pm\infty$$

$\Delta\Psi_0 = +\infty$ için $2M_0^2A > kT$

$M_0 > \pm\left(\frac{kT}{2A}\right)^{\frac{1}{2}}$



$$\frac{\Delta M}{T_c^{\frac{1}{2}}} = \left(\frac{k}{2A}\right)^{\frac{1}{2}}$$

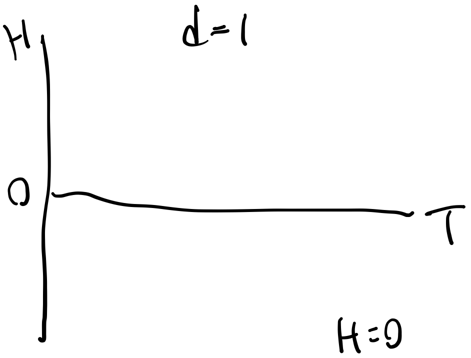
Evrensellik

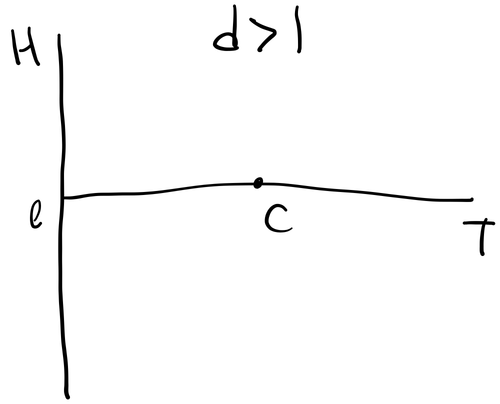
$\frac{\Delta \rho s}{T_c} \to$ evrensel

Potansiyelin uzun menzilliliğine bağlı

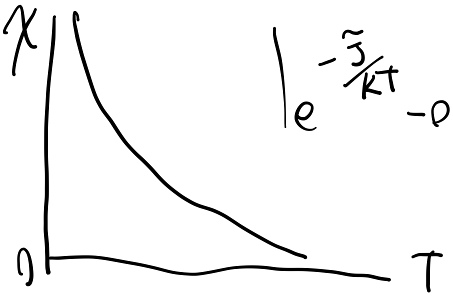
$$f = -kT\ln\lambda_> = -kT\ln\left(e^{\hat{J}/kT}\cosh H + \left(e^{2\hat{J}/kT}\sinh^2 H + e^{-2\hat{J}/kT}\right)^{\frac{1}{2}}\right)$$

$T \to 0$, $e^{-\hat{J}/kT}$ tekillik veriyor.

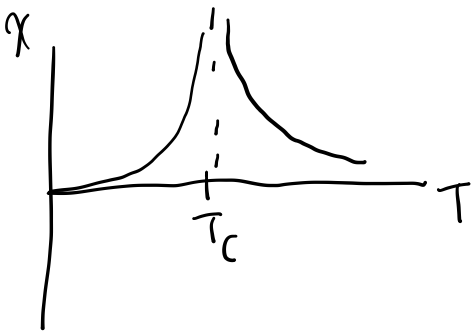




H=0



$$\left|e^{-\hat{J}/kT} - 0\right|^{-\gamma} \qquad \gamma = 2$$



$$\chi = \frac{\partial M}{\partial H} \sim \frac{\partial^2 \ln\lambda_>}{\partial H^2}$$

$T \to 0$

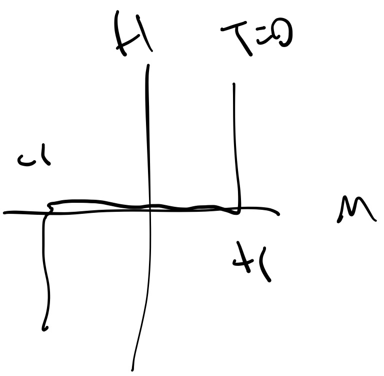
$\hat{J} \to \infty$

$$\lambda_\pm = e^{J}(\cosh H \pm \sinh H) = e^{J \pm H}$$

$$J = \frac{\hat{J}}{kT}$$

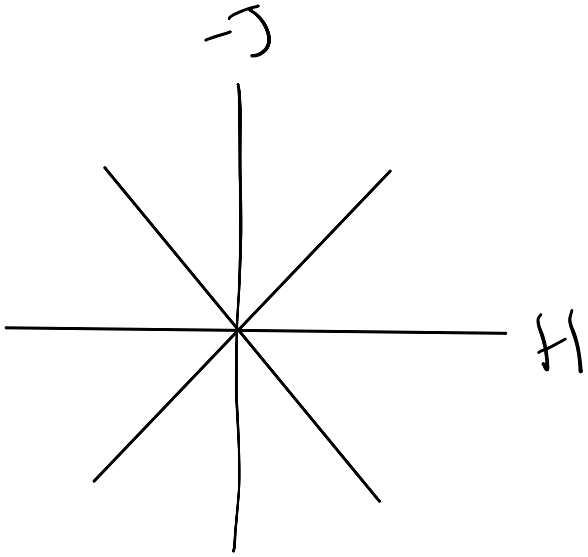
$H > 0 \Rightarrow \ln\lambda_> = J + H$

$$M = \frac{\partial \ln\lambda_>}{\partial H} = 1$$

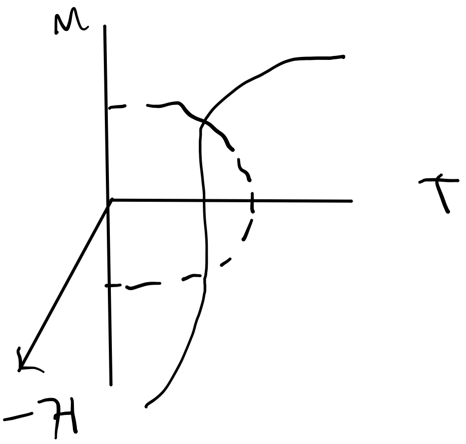


$H < 0 \Rightarrow \ln\lambda_> = J - H$

$$M = \frac{\partial \ln\lambda_>}{\partial H} = -1$$



$$\langle s_1 \rangle = \frac{1}{Z}\sum_{\{s\}} s_1 e^{\left(\frac{H}{2}s_1 + J s_1 s_2 + \frac{H s_2}{2}\right)} \cdots$$



$$M(N) = \frac{\sum_{\{s\}} \left(\frac{1}{N}\sum_i s_i\right) e^{J\sum_{\langle ij\rangle} s_i s_j}}{\sum_{\{s\}} e^{J\sum_{\langle ij\rangle} s_i s_j}}$$

$$s' \equiv -s$$

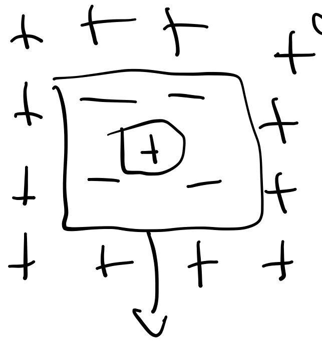
$$\frac{\sum_{\{s'\}} \left(\frac{1}{N}\sum_i [-s_i'] e^{J\sum_{\langle ij\rangle} s_i' s_j'}\right.}{\sum_{\{s\}} e^{J\sum_{ij} s_i' s_j'}} = -M(N)$$

$$\Longrightarrow M(N) = 0$$

$$\lim_{N\to\infty} \lim_{H\to 0} M(N,H,T<T_c) = 0$$

$$\lim_{H\to 0} \lim_{N\to 0} M(N,H,T<T_c) = M_0(T) > 0$$

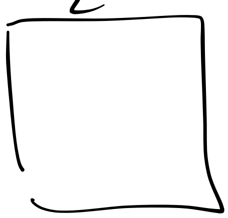
dış çevre



I eksi adası

$$P_b(I) \begin{cases} \text{ada var} & I \\ \text{yok} & 0 \end{cases}$$

$\frac{b}{2} - x$



x

$$\frac{d}{d} N_- = \frac{d}{dx}\left(\frac{b}{2} - x\right)x = \left(\frac{b}{2} - x\right) - x = 0$$

$$x_{max} = \frac{b}{4}$$

$$H_{-} \leq \sum_{b=4,6,8} \left(\frac{b}{4}\right)^2 \sum_{I=1}^{n_b} P_b(I)$$

$$\langle P_b(I) \rangle = \frac{\sum_{\{s\}} e^{-\beta \mathcal{H}} P_b(I)}{\sum_{\{s\}} e^{-\beta \mathcal{H}}} = \frac{\sum' e^{-\beta \mathcal{H}}}{\sum e^{-\beta \mathcal{H}}}$$

rasgu

$$\text{üst limit} \leq \frac{\sum' e^{-\beta \mathcal{H}}}{\sum'' e^{-\beta \mathcal{H}}} = e^{-2bJ}$$

n_b öteleme $\leq N$

şekiller

$$N_- \leq \sum \left(\frac{b}{4}\right)^2 \sum_{I=1}^{n_b} P_b(I)$$

$b = 4, 6, 8 \ldots$

yalnız I varsa

$$\langle P_b(I)\rangle = \frac{\sum_{\{s\}} P_b(I) e^{-\beta \mathcal{H}}}{\sum_{\{s\}} e^{-\beta \mathcal{H}}} = \frac{\sum' e^{-\beta \mathcal{H}}}{\sum e^{-\beta \mathcal{H}}} = \frac{\sum' e^{-\beta \mathcal{H}}}{\sum'' e^{-\beta \mathcal{H}}} = e^{-2bJ}$$

tersyüz edilmiş

n_b: # öteleme $\leq N$

şekiller $\leq 4 \cdot 3^{b-2}$ $\Longrightarrow N_- \leq \sum_b \left(\frac{b}{4}\right)^2 N \cdot 4 \cdot 3^{b-2} e^{-2Jb}$

$$= \frac{N}{36} \sum_{b=1}^{\infty} b^2 \left(e^{-2J + \ln 3}\right)^b$$

$$\frac{\langle N_- \rangle}{N} = \frac{1}{36} \sum_{b=1}^{\infty} b^2 \left(e^{-2J + \ln 3}\right)^b$$

108 36.6

$6\sqrt{6}$

$$S(y) = \sum_{b=0}^{\infty} (e^{-y})^b \underset{\text{geo. serie}}{=} \frac{1}{1-e^{-y}}$$

$$S'(y) = -\sum_{b=1}^{\infty} b (e^{-y})^b = -\frac{e^{-y}}{(1-e^{-y})^2} = -\left(e^{\frac{y}{2}} - e^{-\frac{y}{2}}\right)^{-2}$$

$$\frac{d^2 S(y)}{dy^2} = \sum_{b=1}^{\infty} b^2 (e^{-y})^b = \frac{e^{\frac{y}{2}} + e^{-\frac{y}{2}}}{\left(e^{\frac{y}{2}} - e^{-\frac{y}{2}}\right)^3} \xrightarrow{\lim_{y\to\infty}} 0$$

$$\frac{\langle N_- \rangle}{N} \leq \frac{1}{36} \sum_{b=1}^{\infty} b^2 \left(e^{\frac{-2J}{kT} + \ln 3} \right)^b < \frac{1}{2} - \frac{|\alpha|}{2}$$

$0 < |\alpha| < 1$

T'yi kafi derecede düşürmek kaydıyla

$$M(N, H=0) = \frac{\langle N_+ \rangle - \langle N_- \rangle}{N} = 1 - \frac{2\langle N_- \rangle}{N} > |\alpha|$$

$M > |\alpha| > 0$

$$Z = \sum_{\{s\}}^{2^N} e^{J \sum_{ij} s_i s_j} = \sum_{\{t\}}^{2^{2N}} e^{J \sum_{\langle ij \rangle} t_{ij}}$$

N tane $\{s_i\}$ varken 2N tane $\{t_{ij}\}$ var $\rightarrow$ uyuşmazlık

$$\underbrace{t_{12}\, t_{23}\, t_{34}\, t_{41}}_{\text{bağımsız değiller}} = s_1^2 s_2^2 s_3^2 s_4^2 = 1$$

N tane şart var

$$Z = \frac{1}{2^N} \sum_{\{t\}} e^{J \sum_{ij} t_{ij}} \left(\prod_p \sum_{n_p = 0,1} (t_{12} t_{23} t_{34} t_{41} + 1)^{n_p} \right)$$

$$= \frac{1}{2^N} \sum_{\{n_p = 0,1\}} \sum_{\{t\}} \left(\prod_{ij} e^{J t_{ij}} \right) \left(\prod_p (t_{12} t_{23} t_{34} t_{41})^{n_p} \right)$$

$$= \frac{1}{2^N} \sum_{\{n_p\}} \sum_{\{t\}} \prod_{\langle ij \rangle} e^{J t_{ij}}\, t_{ij}^{\,n_p + n_r}$$

$$= \frac{1}{2^N} \sum_{\{n_p\}} \prod_{ij} \sum_{t_{ij} = \pm 1} e^{J t_{ij}}\, t_{ij}^{\,n_p + n_r}$$

$$= \frac{1}{2^N} \sum_{\{n_p\}} \prod_{ij} \left[e^{J} + (-1)^{n_p+n_r} e^{-J} \right]$$

$$\left. \begin{aligned} \sigma_p &= 2n_p - 1 \\ n_p &= \frac{1}{2}(\sigma_p + 1) \end{aligned} \right\} \text{0,1 dönüşüm}$$

$$Z = \frac{1}{2^N} \sum_{\{\sigma_p\}} \prod_{\{pr\}} \left[e^{J} - (-1)^{\frac{\sigma_p+\sigma_r}{2}} e^{-J} \right]$$

$$e^{J} - (-1)^{\frac{(\sigma_p+\sigma_r)}{2}} e^{-J} = e^{\tilde{J}\sigma_p\sigma_r + \tilde{G}}$$

$$+1,+1 = e^{-J} + e^{J} = e^{\tilde{J}+\tilde{G}}$$

$$+1-1 = e^{J} - e^{-J} = e^{-\tilde{J}+\tilde{G}}$$

$$\tanh(J) = e^{-2\tilde{J}}$$

$$Z(J) = \frac{1}{2^N} \sum_{\{\sigma_p\}} \prod_{\{pr\}}^{2N} \left(2\sinh(2J)\right)^{1/2} e^{\tilde{J}\sigma_p\sigma_r}$$

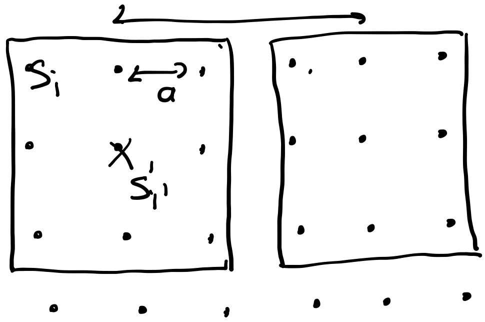
$$= \left(\sinh(2J)\right)^{N} \sum_{\{pr\}} e^{\sum_{\{pr\}} \tilde{J}\sigma_p\sigma_r}$$

Kadanoff:

$$-\beta \mathcal{H} = J \sum_{\langle ij \rangle} s_i s_j + H \sum_i s_i$$

Kritik nokta: $J = J_c$, $H = H_c = 0$

$$t = \frac{J_c - J}{J_c}, \quad H$$



Hücre serbesti derecesi $s'_{i'}$

$$S'_{i'} = b^{-d} \sum_{i}^{i'} s_i$$

$$s'_{i'} = \mathrm{sign}\left(\sum_{i}^{i'} s_i \right) = \pm 1$$

s problemi

N spin

en yakın komşu a

$$-\beta \mathcal{H} = J \sum_{\langle ij \rangle} s_i s_j + H \sum_i s_i$$

t, H

s' problemi

$$N' \text{ spin} = \frac{N}{b^d}$$

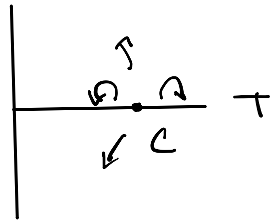
en yakın komşu $a' = ba$

$$-\beta' \mathcal{H}' = J' \sum_{\langle ij \rangle'} s'_{i'} s'_{j'} + H' \sum_{i'} s'_{i'}$$

t', H'

$$Z(t,H) = Z(t',H')$$

$$\left.\begin{aligned} t' &= t'(t,H) \\ H' &= H'(t,H) \end{aligned}\right\} \text{Tekrarlama bağıntıları}$$



$t' > t > 0$ or $0 > t > t'$

$H' > H > 0$ or $0 > H > H'$

$t'(t=H=0 \text{ ise}) = 0, \quad H'(t=H=0 \text{ ise}) = 0, \quad J'(J=J_c, H=0 \text{ ise}) = J_c$

$t = H = 0$ etrafında açılım.

$t' = At + BH$

$H' = Ct + DH$

$$\underbrace{\left.\begin{aligned} s &\longrightarrow -s \\ s' &\longrightarrow -s' \end{aligned}\right\} \begin{aligned} H &\longrightarrow -H \\ H' &\longrightarrow -H' \\ t &\longrightarrow t \\ t' &\longrightarrow t' \end{aligned} \Longrightarrow \begin{aligned} B &= 0 \\ C &= 0 \end{aligned}}$$

$$t' = b^{y_T} t$$

$$H' = b^{y_H} H$$

$b = 1 \Rightarrow t' = t, \; H' = H$ and $b_{12} = b_1 b_2$

Serbest Enerji

$\ln Z = \ln Z' \quad , \quad f = \frac{1}{N} \ln Z(t,H)$

$N f(t,H) = N' f(t',H')$

$f(t,H) = b^{-d} f(b^{y_T} t, b^{y_H} H) \longrightarrow$ Genelleştirilmiş Homojen Fonksiyon

$b = t^{-1/y_T}$ seçelim : $f(t,H) = t^{d/y_T} f\left(1, \frac{H}{t^{y_H/y_T}}\right)$

ölçeklenme şekli

$2-\alpha = \frac{d}{y_T}$, $\frac{d}{y_T} - 1 \geq 0$

$d \geq y_T > 0$

$b = H^{-y_H}$ seçelim $f(t,H) = H^{d/y_H} f\left(\frac{t}{H^{y_T/y_H}}, 1\right)$

$\frac{d}{y_H} - 2 = -\gamma$; $\frac{d}{y_H} - 1 \geq 0$

$\frac{2y_H - d}{y_T}$ $\qquad d \geq y_H$

Bağlantı uzunluğu $H=0$ için

$$\xi = \xi_0 t^{-\nu}, \quad \xi' = \frac{\xi}{b} = \xi_0 t'^{-\nu} = \xi_0 b^{-y_T \nu} t^{-\nu} = \frac{\xi_0 t^{-\nu}}{b} \Rightarrow \nu = \frac{1}{y_T}$$

$t=0$ için

$$\xi = \xi_0 H^{-\nu_H}, \quad \nu_H = \frac{1}{y_H}$$

$$M(t,H) = \frac{1}{N}\frac{\partial}{\partial H} \ln Z(t,H) = \frac{1}{b^d N'} \frac{\partial}{b^{-y_H}\frac{\partial}{\partial H}} \ln Z(t',H') = b^{y_H - d} M(t',H')$$

$b = t^{-\frac{1}{y_T}}$ seçelim : $M(t,H) = t^{\frac{d-y_H}{y_T}} M\left(1, \frac{H}{t^{y_H/y_T}}\right)$

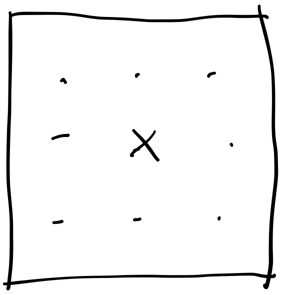
$$\beta = \frac{d-y_H}{y_T}$$

$b = H^{-\frac{1}{y_H}}$ seçelim : $M(t,H) = H^{\frac{d-y_H}{y_H}} M\left(\frac{t}{H^{\frac{y_T}{y_H}}}, 1\right)$

$$M \sim H^{1/\delta}$$

$$\delta = \frac{y_H}{d-y_H}$$

Bağlantı Fonksiyonu:



Düzgün manyetik alan : $H' = b^{y_H} H$

Yerel manyetik alan : $H_i' = \frac{b^{y_H}}{b^d} H_i$

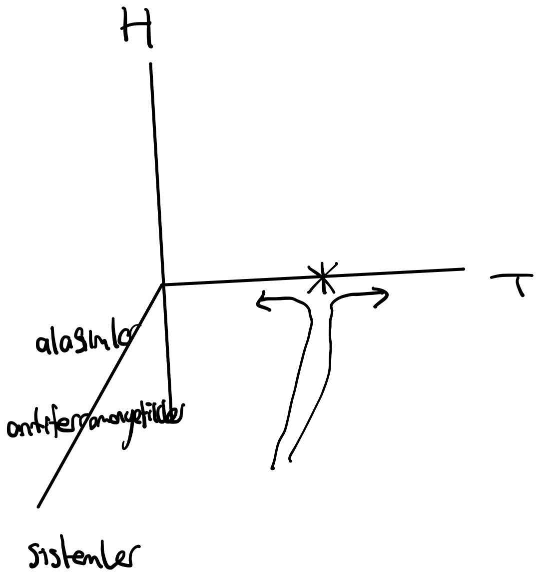
$$\Gamma(r,t,H) = \langle s_0 s_r \rangle - \langle s_0 \rangle \langle s_r \rangle = \frac{\partial^2}{\partial H_0 \partial H_r} \ln Z = b^{2y_H - 2d} \frac{\partial^2}{\partial H_0' \partial H_r'} \ln Z$$

$$= b^{2y_H - 2d} \Gamma(r', t', H')$$

$b = r$ seçersek $\Gamma(r,t,H) = \frac{\Gamma(1, r^{y_T} t, b^{y_H} H)}{r^{2d-2y_H}}$

$$\Gamma(r,t,H) = \frac{\text{fonks}(r^{y_T} t, r^{y_H} H^r)}{r^{2d-2y_H}}$$

$$v = 2d - 2y_H = d - 2 + \eta \implies \eta = d + 2 - 2y_H$$



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d=1 Ising Modeli

$$Z = \sum_{\{s\}} e^{-\beta \mathcal{H}\{s\}} = \sum_{\{s'\}} \left(\sum_{\{\sigma\}} e^{-\beta \mathcal{H}\{s', \sigma\}} \right) \overset{RG}{=} \sum_{\{s'\}} e^{-\beta' \mathcal{H}'\{s'\}}$$

$i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$i' = 1 \quad 2 \quad 3$

$$b = 2 \qquad S'_{i'} = S_{i = 2i'}, \quad \sigma_{i'} = S_{2i'-1}$$

$$\sum_{S_3} e^{J s_2 s_3 + H s_3 + J s_3 s_4} = e^{J' s_2 s_4 + \tilde{H}(s_2 + s_4) + \tilde{G}} \equiv R(s_2, s_4)$$

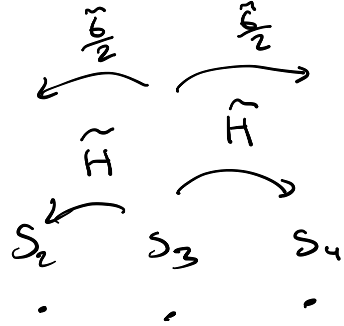
$$x = e^J, \quad y = e^H, \quad \tilde{g} = e^{\tilde{G}}$$

$$R(+,+) = x^2 y + x^{-2} y^{-1} = x' \tilde{y}^2 \tilde{g}$$
$$R(+,-) = y + y^{-1} = x'^{-1} \tilde{g}$$
$$R(-,-) = x^2 y^{-1} + x^{-2} y = x' \tilde{y}^{-2} \tilde{g}$$

$$x'^4 = \frac{R(++)R(--)}{R(+-)R(-+)} \qquad \tilde{y} = \frac{R(++)}{R(--)}$$

$$\tilde{g}^4 = R(++)R(--)R(+-)R(-+)$$

$$x'^4 = \frac{(x^2 y + x^{-2} y^{-1})(x^2 y^{-1} + x^{-2} y)}{(y+y^{-1})^2}$$



$$H' = H + 2\tilde{H}$$

$$y'^2 = y_2 \frac{x^2 y + x^{-2} y^{-1}}{x^2 y^{-1} + x^{-2} y}$$

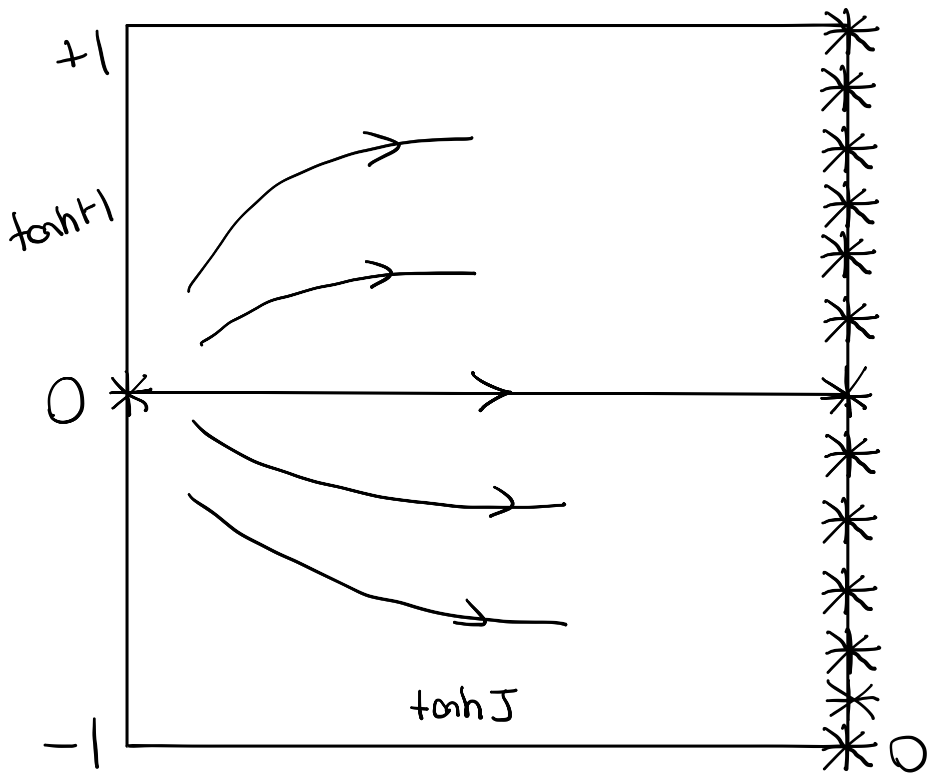
$$G' = 2G + \tilde{G}$$

$$g'^4 = g^8 (x^2 y + x^{-2} y^{-1})(x^2 y^{-1} + x^{-2} y)$$

$$J' = J'(J,H)$$
$$H' = H'(J,H)$$
$$G' = b^d G + \tilde{G}(J,H)$$

$$H = 0 \Rightarrow H' = 0$$

$$\left.\begin{array}{l} s, s' \longrightarrow -s, -s' \\ H, H' \longrightarrow -H, -H' \\ J, J' \longrightarrow J, J' \end{array}\right\} \quad H=0 \text{ için} \quad \tanh J' = \{\tanh(J)\}^2$$



$H^* = 0$, $J^* = 0$ kararlı

$H^* = 0$, $J^* = \infty$ kararsız

Bağlantı uzaklığı

$\xi = fonk(J, H)$ $\quad \xi' = fonk(J', H')$ $\quad$ ve $\quad \xi = b\,\xi'$

Sabit noktada: $J = J' = J^*$, $H = H' = H^* = 0$

$\xi = \xi'$ ise $\quad \xi = 0$ veya $\xi = \infty$

$$\tanh(J') = [\tanh(J)]^2$$

$$\underbrace{1 - 2e^{-2J'} \simeq \left(1 - 2e^{-2J}\right)^2 = 1 - 4e^{-2J}}_{e^{-J'} = 2^{\frac{1}{2}} e^{-J}}$$

$$\Downarrow$$

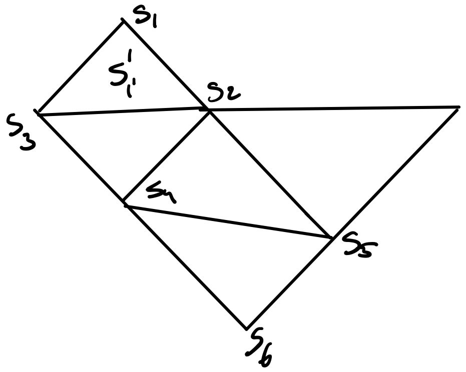
$t' = b^{y_T} t$ eşitliğinden $\quad b = 2$, $\quad y_T = \frac{1}{2}$, $\quad \nu = \frac{1}{y_T} = 2$

$H' = 2H$

$H' = b^{y_H} H$ $\quad \Longrightarrow \quad y_H = 1 \quad \Longrightarrow \quad \gamma = \dfrac{(2y_H - d)}{y_T} = 2$

$$\chi \sim e^{-J^2}$$

$$\eta = d + 2 - 2y_H = 1$$



$$\frac{N}{N'} = b^d = 3 \Rightarrow b = \sqrt{3}$$

$$s'_{1'} = \mathrm{sign}(s_1+s_2+s_3) = \pm 1$$

$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}\{s\}} = \sum_{\{s'\}}\left(\sum_{\{\sigma\}} e^{-\beta\mathcal{H}\{s'\sigma\}}\right) = \sum_{\{s'\}} e^{-\beta'\mathcal{H}'\{s'\}} = Z'$$

$$R(s'_{1'}, s'_{2'}) = e^{J' s'_1 s'_2 + \tilde{G}} = \sum_{\{\sigma\}} e^{J(s_1s_2 + s_2s_3 + s_3s_1 + s_4s_5 + s_5s_6 + s_6s_4 + s_2s_4 + s_3s_4)}$$

$$e^{2J'} = \frac{R(++)}{R(+-)}, \quad e^{2\tilde{G}} = R(++)R(+-)$$

$R(++) =$ polinom, $x = e^J$ iken $x^?$ üsteller

$R(+-) =$ polinom, $x = e^J$ iken $4 \leq$ üsteller

$R(++)$	$3J$ △ (+; + +)	$-J$ △ (−; + +)	$-J$ △ (+; + −)	$-J$ △ (+; − +)	$R(-+)$
△ (+; + +) $3J$	$+2J \to 8J$ $-2J \to 4J$	$+2J \to 4J$ $-2J \to 0$	$0 \to 2J$ $0 \to 2J$	$0 \to 2J$ $0 \to 2J$	△ (−; − −) $3J$
△ (−; + +) $-J$	$-2J \to 0$ $+2J \to 4J$	$-2J \to -4J$ $+2J \to 0$	$0 \to -2J$ $0 \to -2J$	$0 \to -2J$ $0 \to -2J$	△ (+; − −) $-J$
△ (+; + −) $-J$	$+2J \to 4J$ $-2J \to 0$	$+2J \to 0$ $-2J \to -4J$	$0 \to -2J$ $0 \to -2J$	$0 \to -2J$ $0 \to -2J$	$\to$ △ (−; − +)
△ (+; − +) $-J$	$+2J \to 4J$ $-2J \to 0$	$+2J \to 0$ $-2J \to -4J$	$0 \to -2J$ $0 \to -2J$	$0 \to -2J$ $0 \to -2J$	$-J$ △ (−; + −)

$$R(++) = x^8 + 3x^4 + 2x^2 + 3 + 6x^{-2} + x^{-4}$$

$$R(+-) = 2x^4 + 2x^2 + 4 + 6x^{-2} + 2x^{-4}$$

$$x'^2 = \frac{R(++)}{R(+-)} \qquad \tilde{g}^2 = R(++)R(+-)$$

$$x \to \infty \quad J \to \infty \quad T \to 0 \implies x'^2 = \frac{x^8}{2x^4} = \frac{1}{2}x^4$$

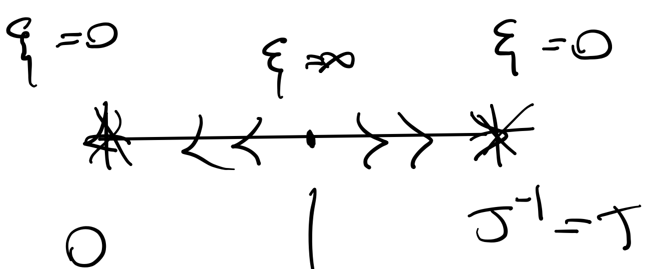
$$J' = 2J > J \qquad J^* = \infty$$ kararlı

$$x \to 1 \quad J \to 0 \quad T \to \infty \implies x = e^J \simeq 1 + J$$

$$x^8 = e^{8J} \simeq 1 + 8J$$

$$x'^2 = \frac{16+8J}{16-8J} \simeq 1 + J = 1 + 2J'$$

$$J' = \frac{J}{2} \qquad J^* = 0$$ kararlı



$$J^* = 0{,}365$$

J^* yakininda

$$J'(J) = J'(J^*) + \left.\frac{\partial J'}{\partial J}\right)_{J^*} (J - J^*)$$

$$J' - J^* = \left.\frac{\partial J'}{\partial J}\right)_{J^*} (J - J^*)$$