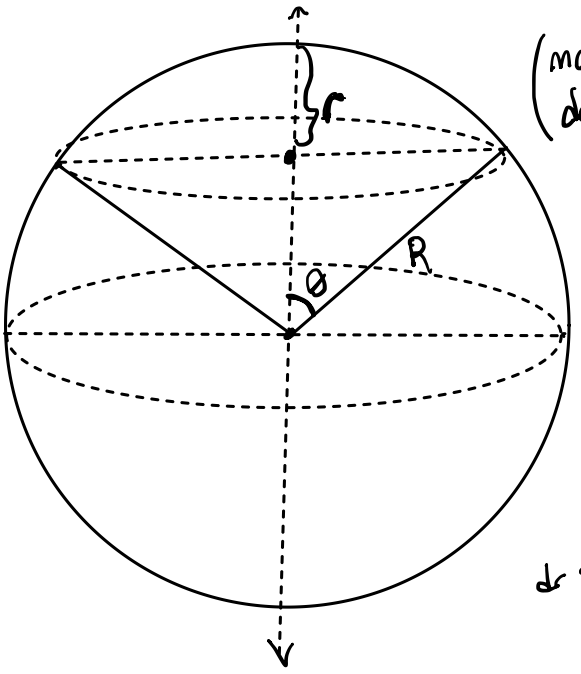
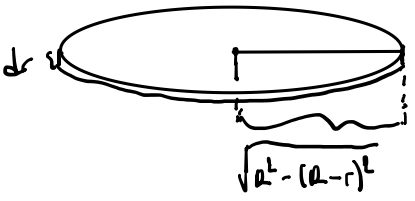


$\left(\begin{matrix}\text{mass}\\ \text{density}\end{matrix} : \rho\right) \quad \left(\rho = \frac{M}{\frac{4}{3}\pi R^3}\right)$

from the I of an uniform circle

$dI = \frac{1}{2} dm \left[R^2 - (R-r)^2\right]$



$\rightarrow dI = \frac{1}{2}\rho\pi\left[R^2-(R-r)^2\right]^2 dr$

$dm = \rho\pi\left[R^2-(R-r)^2\right]dr$

$$\int dI = \int_0^{2R} \frac{1}{2}\rho\pi\left[R^2-(R-r)^2\right]^2 dr = \frac{1}{2}\cdot\rho\cdot\pi\cdot\frac{16R^5}{15} = \frac{1}{2}\cdot\frac{3M}{4\pi R^3}\cdot\pi\cdot\frac{16R^5}{15} = \frac{2}{5}MR^2$$