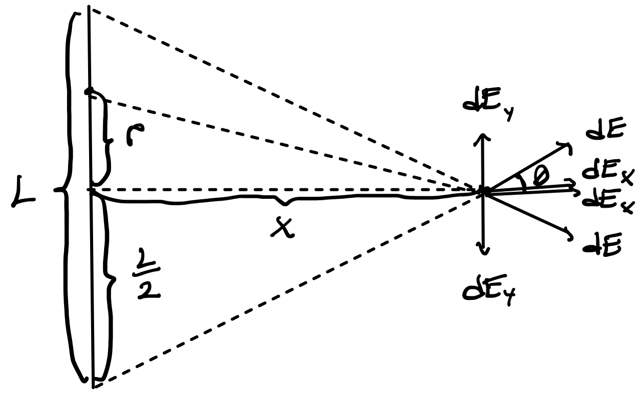


$\left(\text{charge density}; \lambda\right) \quad \left(\lambda = \frac{Q}{L}\right)$

dE_y's cancel out due to symmetry

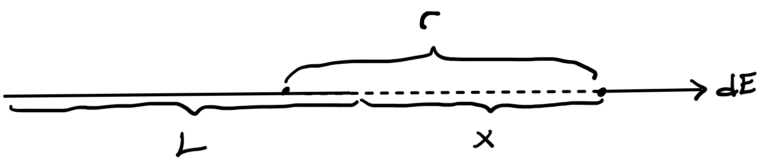
$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x^2+r^2)}\frac{x}{\sqrt{x^2+r^2}}\hat{x}$$

$$dq = \lambda dr$$

$$\rightarrow dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+r^2)^{\frac{3}{2}}}dr\,\hat{x}$$

$$\int dE_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(x^2+r^2)^{\frac{3}{2}}}\hat{x}\,dr = \frac{1}{4\pi\varepsilon_0}\lambda x\frac{2L}{x^2\sqrt{4x^2+L^2}}\hat{x} = \frac{1}{2\pi\varepsilon_0}\frac{L}{x\sqrt{4x^2+L^2}}\cdot\frac{Q}{L}\hat{x} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{x\sqrt{4x^2+L^2}}\hat{x} = \vec{E}$$

$$= \frac{2kQ}{x\sqrt{4x^2+L^2}}\hat{x} = \vec{E}$$



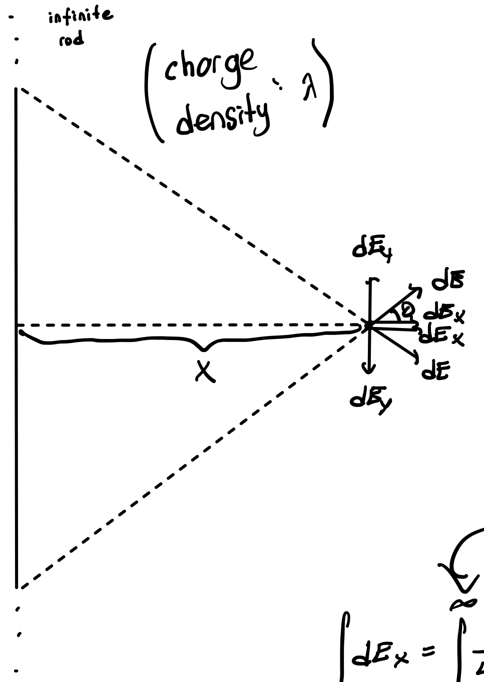
$\left(\text{charge density}, \lambda\right) \quad \left(\lambda = \frac{Q}{L}\right)$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{x}$$

$$dq = \lambda dr$$

$$\rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}dr\,\hat{x}$$

$$\int dE = \int_{x}^{x+L}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\hat{x}\,dr = \frac{1}{4\pi\varepsilon_0}\lambda\cdot\frac{L}{x(x+L)}\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{L}{x(x+L)}\cdot\frac{Q}{L}\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x(x+L)}\hat{x} = \frac{kQ}{x(x+L)}\hat{x} = \vec{E}$$



dE_y's still cancel out

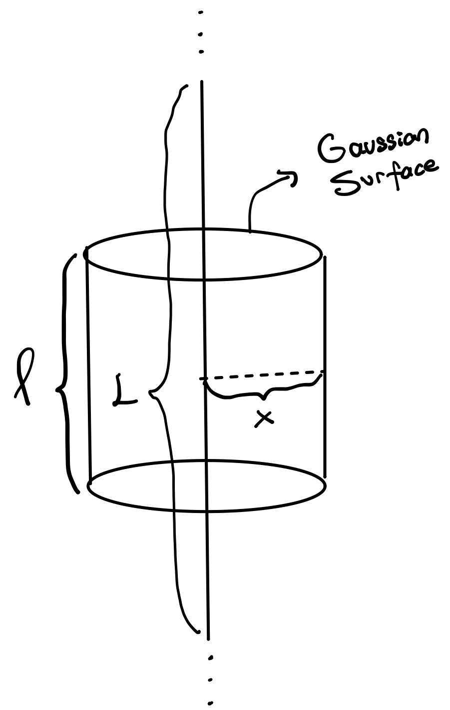
$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{(x^2+r^2)}\frac{x}{\sqrt{x^2+r^2}}\hat{x}$$

$$dq = \lambda dr$$

$$\rightarrow dE_x = \frac{1}{4\pi\varepsilon_0}\frac{x\lambda}{(x^2+r^2)^{\frac{3}{2}}}dr\,\hat{x}$$

$$\int dE_x = \int_{-\infty}^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{x\lambda}{(x^2+r^2)^{\frac{3}{2}}}\hat{x}dr = \frac{1}{4\pi\varepsilon_0}x\lambda\cdot\frac{2}{x^2}\hat{x} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{x} = \frac{2k\lambda}{x}\hat{x} = \vec{E}$$

OR



$$\underbrace{\underset{\text{upper cab}}{\oint}\vec{E}.d\vec{A} + \underset{\text{lower cab}}{\oint}\vec{E}.d\vec{A}}_{0} + \underset{\text{curved surface}}{\oint}\vec{E}.d\vec{A} = \underset{\text{curved surface}}{\oint}\vec{E}.d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

0 since the angle between $\vec{E}$ and their normal line is $90° \Rightarrow \cos 90° = 0$

(assume that it has a length L)

$$|\vec{E}|\oint d\vec{A} = |\vec{E}|.2\pi x l = \frac{|Q| l}{L\varepsilon_0} \rightarrow |\vec{E}| = \frac{1}{2\pi\varepsilon_0}\frac{|Q|}{xL}$$

since dE_y's cancel out

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{xL}\hat{x} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{x} = \frac{2k\lambda}{x}\hat{x}$$

$\vec{E}$ is constant since it always points radially outward (which forms a 0° angle between $\vec{E}$ and the normal line) and the rod is uniformly charged (which makes $|\vec{E}|$ constant)