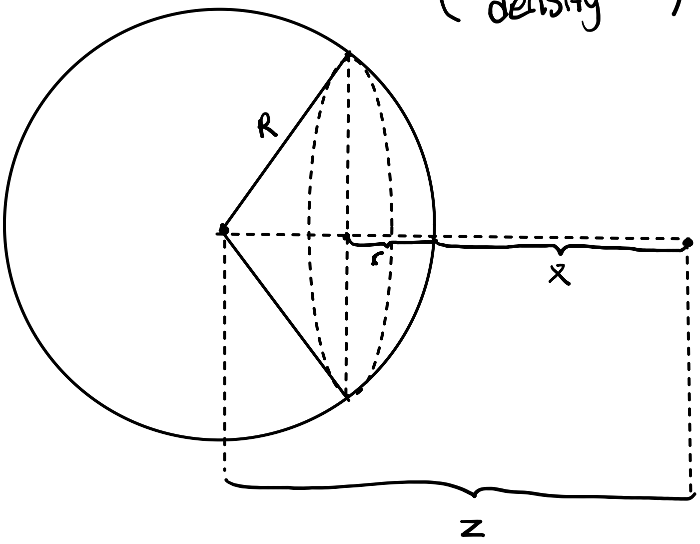
$$\begin{pmatrix}\text{charge} \\ \text{density}\end{pmatrix} : \rho \qquad \left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right)$$



from the $\vec{E}$ of a uniformly charged circle

$$dE_x = \frac{1}{2\pi\varepsilon_0}\frac{dq}{R^2-(R-r)^2}\left(1-\frac{x+r}{\sqrt{(x+r)^2+R^2-(R-r)^2}}\right)\hat{x}$$

$$dq = \rho\, dV$$

$$dV = \pi\left[R^2-(R-r)^2\right]dr$$

$$\rightarrow dE_x = \frac{\rho}{2\varepsilon_0}\left(1-\frac{x+r}{\sqrt{(x+r)^2+R^2-(R-r)^2}}\right)dr\,\hat{x}$$

$$\int dE_x = \int_0^{2R}\frac{\rho}{2\varepsilon_0}\left(1-\frac{x+r}{\sqrt{(x+r)^2+R^2-(R-r)^2}}\right)\hat{x}\,dr = \frac{\rho}{2\varepsilon_0}\cdot\frac{2R^3}{3(x+R)^2}\hat{x} = \frac{Q}{\frac{4}{3}\pi R^3}\cdot\frac{R^3}{3\varepsilon_0(x+R)^2}\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x+R)^2}\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}\hat{x} = \vec{E}$$

$$= \frac{kQ}{z^2}\hat{x} = \vec{E}$$