KEPLER'S LAWS

First Law: Planets orbit stars in ellipses with stars on foci.

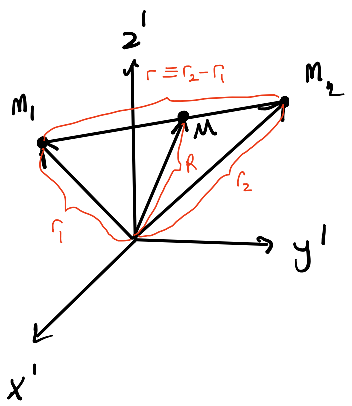
Second Law: A line connecting the sun and the planet sweeps equal areas in equal time intervals.

Third Law: $T^2 \propto a^3$ (T = orbit period, a = average distance of the planet from the sun)

for uniform circular motion: $T = \frac{2\pi r}{v} \Rightarrow \frac{4\pi^2 r^2}{v^2} = kr^3 \Rightarrow \frac{4\pi^2 m}{kr^2} = \frac{mv^2}{r} = F \xRightarrow[\text{reaction law}]{\text{from action}} F = \frac{4\pi^2 m}{kr^2} = \frac{4\pi^2 M}{k' r^2} = \frac{4\pi^2 mM}{k'' r^2} \xRightarrow[G \equiv \frac{4\pi^2}{k''}]{} \frac{GMm}{r^2} = F$

proportionality constant

Center of Mass Reference Frame



$$r \equiv r_2 - r_1$$

$$R \equiv \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \xrightarrow[\text{terms}]{\text{in general}} \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i}$$ for n celestial body

$$\sum_{i=1}^{n} m_i R = MR = \sum_{i=1}^{n} m_i r_i \Longrightarrow M\frac{dR}{dt} = \sum_{i=1}^{n} m_i \frac{dr_i}{dt} = MV \equiv P = \sum_{i=1}^{n} P_i \Rightarrow \frac{dP}{dt} = \sum_{i=1}^{n} \frac{dP_i}{dt}$$

$$F = \frac{dP}{dt} = M\frac{d^2R}{dt^2} = 0$$

If we assume that *all* of the forces acting on individual particles in the system are due to other particles contained within the system, Newton's third law requires that the total force must be zero. This constraint exists because of the equal magnitudes and opposite directions of action–reaction pairs. Of course, the momentum of individual masses may change. Using center-of-mass quantities, we find that the total (or net) force on the system is

Therefore; in this case, center of mass frame is inertial reference frame.

By Newton's second law

$$\vec F_{\text{on planet}} = m_p \ddot{\vec r}_p = -\frac{G m_p m_s}{r^3}\vec r \Longrightarrow \ddot{\vec r}_p = \frac{-G m_s}{r^3}\vec r$$

$$\vec F_{\text{on sun}} = m_s \ddot{\vec r}_s = \frac{G m_p m_s}{r^3}\vec r \Longrightarrow \ddot{\vec r}_s = \frac{G m_p}{r^3}\vec r$$

$$\ddot{\vec r} = \ddot{\vec r}_p - \ddot{\vec r}_s = -\frac{G}{r^3}(m_s + m_p)\vec r = -\mu \frac{1}{r^3}\vec r$$

$$(\mu \equiv G(m_s + m_p))$$

$\vec L$ is constant since no T is exerted on the celestial object.

$$\vec L = m_p \vec r \times \dot{\vec r}$$

$$\vec k \equiv \vec r \times \dot{\vec r} \Longrightarrow \dot{\vec k} = \dot{\vec r} \times \dot{\vec r} + \vec r \times \ddot{\vec r} = \vec r \times \ddot{\vec r} = \vec r \times \left(-\mu \frac{1}{r^3}\vec r\right) = \vec 0 \Longrightarrow \vec k \text{ is a constant vector}$$

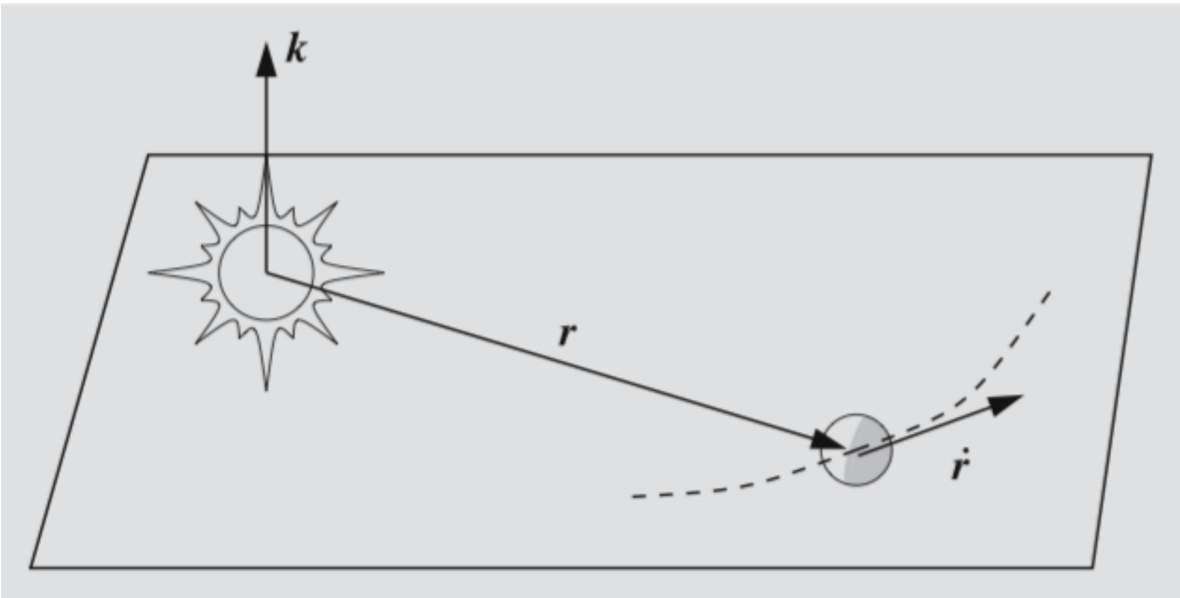


Fig. 6.2. The angular momentum vector $\boldsymbol{k}$ is perpendicular to the radius and velocity vectors of the planet. Since $\boldsymbol{k}$ is a constant vector, the motion of the planet is restricted to the plane perpendicular to $\boldsymbol{k}$