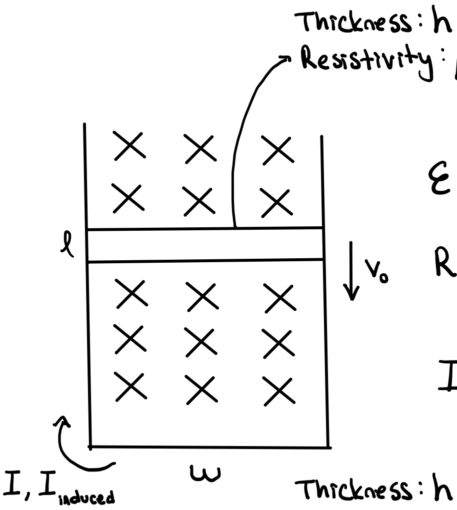
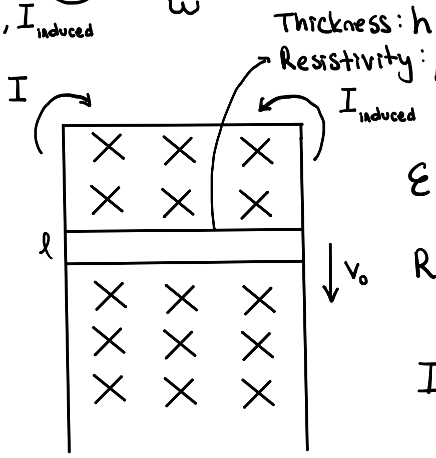



$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Bw\frac{dx}{dt} = Bwv_0$$

$$R = \frac{\rho d}{A_{\substack{cross\\sectional}}} = \frac{\rho w}{h l}$$

$$I_{induced} = \frac{\mathcal{E}_{induced}}{R} = \frac{Bhlv_0}{\rho}$$

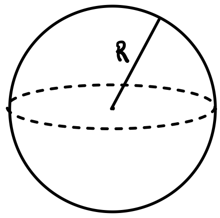



$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Bw\frac{dx}{dt} = -Bwv_0$$

$$R = \frac{\rho d}{A_{\substack{cross\\sectional}}} = \frac{\rho w}{h l}$$

$$I_{induced} = \frac{\mathcal{E}_{induced}}{R} = -\frac{Bhlv_0}{\rho}$$

Uniform Insulating Sphere with charge Q



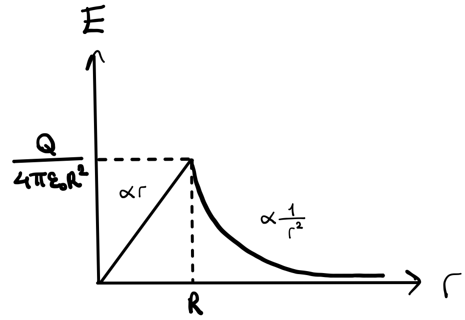

$$\oint \vec{E}\,d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

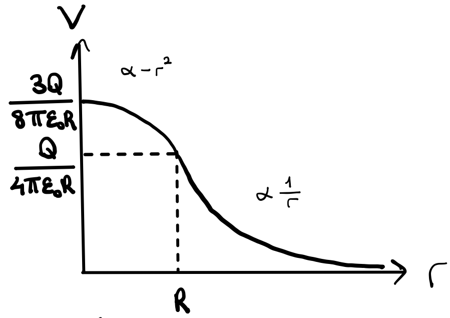
$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3} \Longrightarrow E = \frac{Qr}{4\pi\varepsilon_0 R^3} \quad (r < R)$$

$$4\pi r^2 E = \frac{Q}{\varepsilon_0} \Longrightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (r \geq R)$$

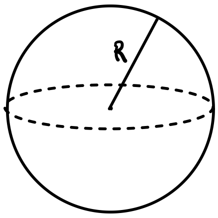
$$V = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0 r} \quad (r \geq R) \Longrightarrow V(R) = \frac{Q}{4\pi\varepsilon_0 R}$$

$$V = \frac{Q}{4\pi\varepsilon_0 R} + \int_r^R \frac{Qr}{4\pi\varepsilon_0 R^3}\,dr = \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3} \quad (r < R)$$




Conducting Sphere with charge Q



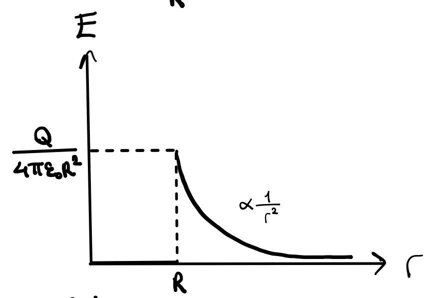

$$\oint \vec{E}\,d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

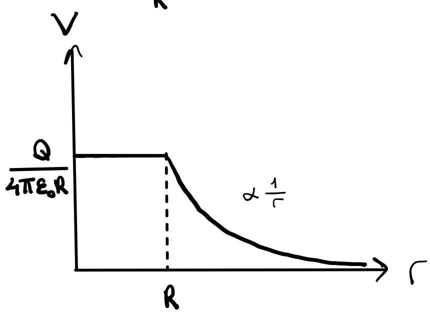
$$E = 0 \quad (r < R)$$

$$4\pi r^2 E = \frac{Q}{\varepsilon_0} \Longrightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (r \geq R)$$

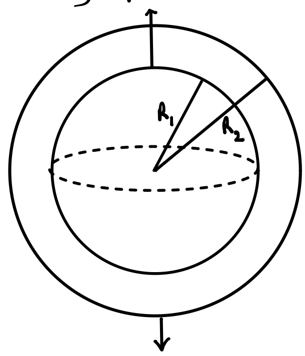
$$V = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0 r} \quad (r \geq R) \Longrightarrow V(R) = \frac{Q}{4\pi\varepsilon_0 R}$$

$$V = \frac{Q}{4\pi\varepsilon_0 R} + \int_r^R 0\,dr = \frac{Q}{4\pi\varepsilon_0 R} \quad (r < R)$$



Conducting Sphere with charge $Q_1 \in \mathbb{R}^+$




Conducting Spherical Shell with charge $Q_2 \in \mathbb{R}^+$

① $$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = 0 \quad (r < R_1)$$

② $$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$4\pi r^2 E = \frac{Q_1}{\varepsilon_0} \Rightarrow E = \frac{Q_1}{4\pi\varepsilon_0 r^2} \quad (R_1 \leq r < R_2)$$

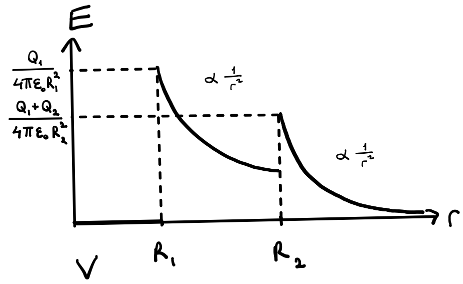
③ $$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

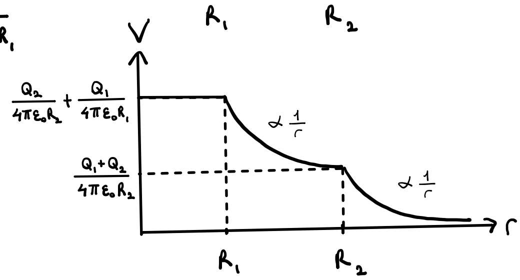
$$4\pi r^2 E = \frac{Q_1+Q_2}{\varepsilon_0} \Rightarrow E = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2} \quad (r \geq R_2)$$

$$V = \int_r^{\infty} \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2} dr = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r} \quad (r \geq R_2) \Longrightarrow V(R_2) = \frac{Q_1+Q_2}{4\pi\varepsilon_0 R_2}$$

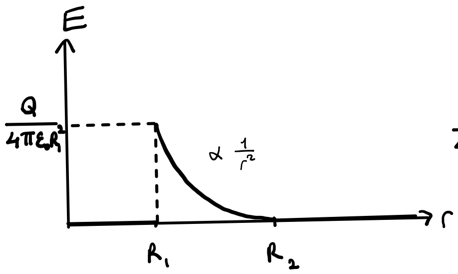
$$V = \frac{Q_1+Q_2}{4\pi\varepsilon_0 R_2} + \int_r^{R_2} \frac{Q_1}{4\pi\varepsilon_0 r^2} dr = \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_1}{4\pi\varepsilon_0 r} \quad (R_1 \leq r < R_2) \Longrightarrow V(R_1) = \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_1}{4\pi\varepsilon_0 R_1}$$

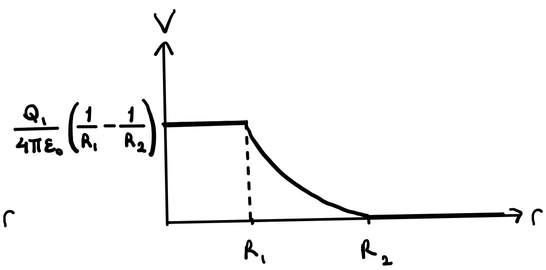
$$V = \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_1}{4\pi\varepsilon_0 R_1} + \int_r^{R_1} 0\, dr = \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_1}{4\pi\varepsilon_0 R_1} \quad (r < R_1)$$

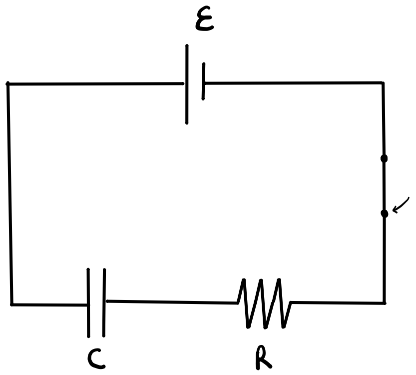



## If the interior sphere has a charge $Q \in \mathbb{R}^+$ and the outer spherical shell was grounded:



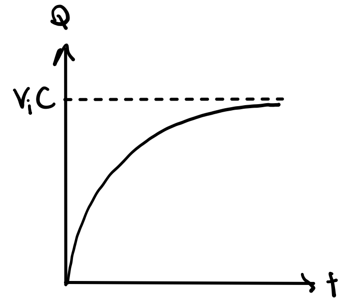

## RC circuit when charging:

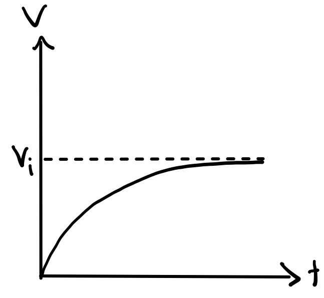
$$\mathcal{E} = V_R + V_C$$

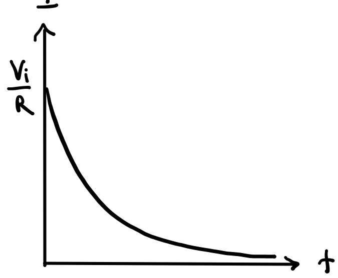
$$Q = VC \Longrightarrow \frac{dQ}{dt} = I_c = I_R = \frac{V_R}{R} = \frac{\mathcal{E} - V_c}{R} = C\frac{dV_c}{dt} \Longrightarrow \frac{dt}{RC} = \frac{dV_c}{\mathcal{E} - V_c}$$

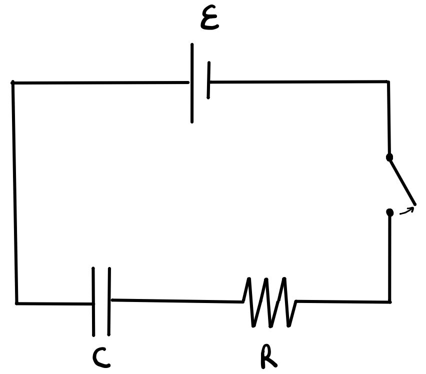
$$\int \frac{dt}{RC} = \int \frac{dV_c}{\mathcal{E} - V_c} \Longrightarrow \frac{t}{RC} + c_1 = -\ln|\mathcal{E} - V_c| \Longrightarrow (\mathcal{E} - V_c)^{-1} = c_2 e^{\frac{t}{RC}} \Longrightarrow \mathcal{E} - V_c = c_3 e^{\frac{-t}{RC}}$$

$$V_c = \mathcal{E} - c_3 e^{\frac{-t}{RC}} \xrightarrow{V_c = 0 \text{ when } t=0} V_c = \mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right) \Longrightarrow CV_c = Q_c = \mathcal{E}C\left(1 - e^{\frac{-t}{RC}}\right) \Longrightarrow \left|\frac{dQ_c}{dt}\right| = I_c = \left|\frac{d\left[\mathcal{E}C\left(1 - e^{\frac{-t}{RC}}\right)\right]}{dt}\right| = \frac{\mathcal{E}}{R} e^{\frac{-t}{RC}}$$





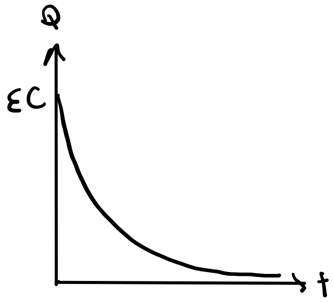

## RC circuit when discharging:

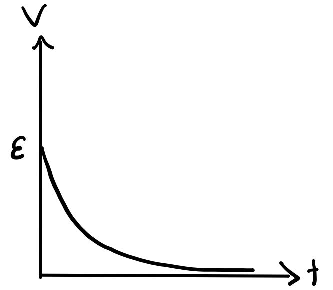
$$V_R + V_C = 0$$

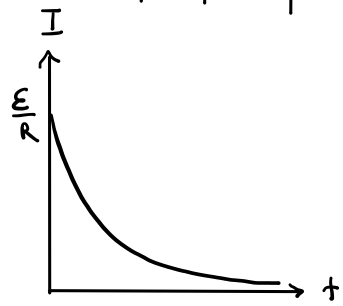
$$Q = VC \Rightarrow \frac{dQ}{dt} = I_c = I_R = \frac{V_R}{R} = \frac{-V_c}{R} = C\frac{dV_c}{dt} \Rightarrow \frac{dt}{RC} = \frac{dV_c}{-V_c}$$

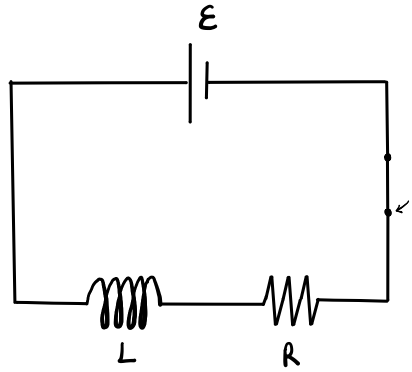
$$\int \frac{dt}{RC} = \int \frac{dV_c}{-V_c} \Rightarrow \frac{t}{RC} + c_1 = -\ln|-V_c| \Rightarrow (-V_c)^{-1} = c_2 e^{\frac{t}{RC}} \Rightarrow -V_c = c_3 e^{\frac{-t}{RC}}$$

$$V_c = -c_3 e^{\frac{-t}{RC}} \xrightarrow{V_c = V_i \text{ when } t=0} V_c = V_i e^{\frac{-t}{RC}} \Rightarrow CV_c = Q_c = CV_i e^{\frac{-t}{RC}} \Rightarrow \left|\frac{dQ_c}{dt}\right| = I_c = \left|\frac{d\left(CV_i e^{\frac{-t}{RC}}\right)}{dt}\right| = \frac{V_i}{R} e^{\frac{-t}{RC}}$$



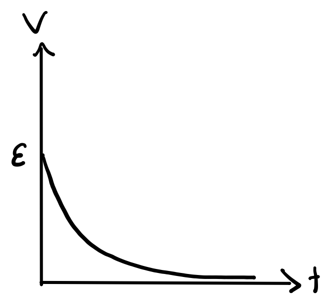

## RL circuit when charging:

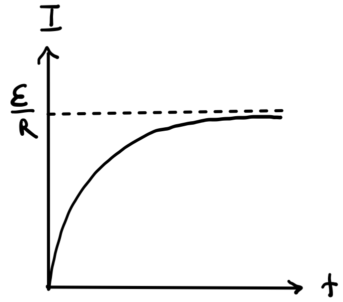
$$\varepsilon = V_R + V_L$$

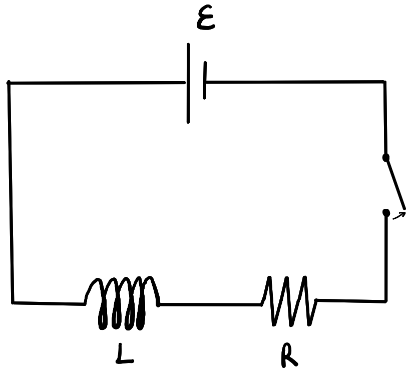
$$V_L = L\frac{dI_L}{dt} = L\frac{dI_R}{dt} = L\frac{d\left(\frac{V_R}{R}\right)}{dt} = L\frac{d\left(\frac{\varepsilon - V_L}{R}\right)}{dt} = -\frac{L}{R}\frac{dV_L}{dt} \Rightarrow -\frac{R}{L}dt = \frac{dV_L}{V_L}$$

$$\int -\frac{R}{L}dt = \int \frac{dV_L}{V_L} \Rightarrow -\frac{Rt}{L} + c_1 = \ln|V_L| \Rightarrow V_L = c_2 e^{-\frac{Rt}{L}}$$

$$V_L = c_2 e^{-\frac{Rt}{L}} \xrightarrow{V_L = \varepsilon \text{ when } t=0} V_L = \varepsilon e^{-\frac{Rt}{L}} \Rightarrow \varepsilon - \varepsilon e^{-\frac{Rt}{L}} = V_R = RI_R = RI_L \Rightarrow I_L = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$





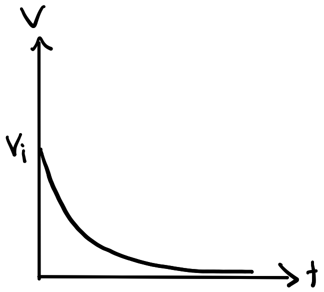

RL circuit when charging:

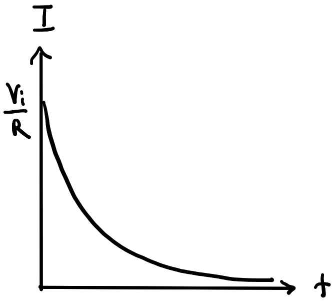
$$V_R + V_L = 0$$

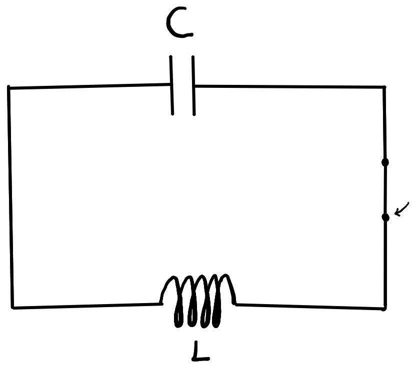
$$V_L = L\frac{dI_L}{dt} = L\frac{dI_R}{dt} = L\frac{d\left(\frac{V_R}{R}\right)}{dt} = L\frac{d\left(\frac{-V_L}{R}\right)}{dt} = -\frac{L}{R}\frac{dV_L}{dt} \Rightarrow -\frac{R}{L}dt = \frac{dV_L}{V_L}$$

$$\int -\frac{R}{L}dt = \int \frac{dV_L}{V_L} \Rightarrow -\frac{Rt}{L} + c_1 = \ln|V_L| \Rightarrow V_L = c_2 e^{-\frac{Rt}{L}}$$

$$V_L = c_2 e^{-\frac{Rt}{L}} \xrightarrow{V_L = V_i \text{ when } t=0} V_L = V_i e^{-\frac{Rt}{L}} \Rightarrow \left|-V_i e^{-\frac{Rt}{L}}\right| = V_R = RI_R = RI_L \Rightarrow I_L = \frac{V_i}{R} e^{-\frac{Rt}{L}}$$




LC circuit when charging:

$$V_c + V_L = 0$$

$$L\frac{dI}{dt} + \frac{Q}{C} = 0 \Rightarrow L\frac{dI}{dt} = L\frac{d^2Q}{dt^2} = -\frac{Q}{C} \Rightarrow \frac{d^2Q}{dt^2} = \frac{-1}{LC}Q$$

Oscillatory Motion Equations : $\frac{d^2\theta}{dt^2} = -\omega^2\theta \Rightarrow \omega = \sqrt{\frac{1}{LC}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$

$$Q_c = Q_i\cos(\omega t) = Q_i\cos\left(\frac{t}{\sqrt{LC}}\right) \Rightarrow \frac{dQ}{dt} = -\frac{Q_i}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right) = I_c \Rightarrow \frac{Q_c}{C} = V_c = \frac{Q_i}{C}\cos\left(\frac{t}{\sqrt{LC}}\right)$$